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Numerical Investigation of Inclination on the Thermal Performance of Porous Fin Heatsink using Pseudospectral Collocation Method

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Numerical Investigation of Inclination on the Thermal Performance of Porous Fin Heatsink using Pseudospectral Collocation Method

Abstract

Numerical investigation of inclination effect on the thermal performance of a porous fin heat sink is presented. The developed thermal model is solved using pseudo-spectral collocation method (PSCM). Parametric studies are carried out using PSCM, and the thermal characterization of heat sink with the inclined porous fin of rectangular geometry is presented. Results show that heat sink of inclined porous fin exhibits higher thermal performance than heat sink of vertical porous fin operating under the same thermal conditions with the same geometrical configurations. Performance of inclined or tilted fin increases with decrease in length-thickness aspect ratio. However, increase in the internal heat generation parameter decreases the fin temperature gradient which invariably decreases the heat transfer performance of the fin. Furthermore, the results of the pseudo-spectral collocation method are compared with the results of Runge-Kutta method. Excellent agreement is established between the results of the non-convectional numerical method and the results of Runge-Kutta, which validates the accuracy of the method for analysis of nonlinear heat transfer problems. The results presented in this paper, hopefully, serves as motivation for the design and optimization of thermally-enhanced heatsinks.

Keywords

Inclination angle, heatsink, porous fin, Pseudospectral collocation method

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1. Introduction

With the continuous advancement in the electronics industry, the demand for high-performance consumer electronics has inadvertently called for their efficient thermal management. High processing performance is usually associated with significant heat dissipation. A direct consequence is often the excess heat build-up within the circuitry of these consumer electronics which usually results in malfunction and their eventual damage. One critical approach to achieve cost-effective, thermally-efficient consumer electronics is the effective heat dissipation between the device surface and its surrounding environment using extended surface or fin. Fin application is identified as a viable approach to enhance the thermal performance of different systems following the research breakthrough of Kiwan et al. [1]. Consequently, research on the phenomena of heat transfer using porous fin has become one emerging research area for engineers and designers. This is because, for equal weight, porous fin has been established to show better performance than solid fins when operating under the same thermal conditions with the same geometrical configurations [2,3].

Extensive research has been carried on porous fin for heat transfer enhancement in thermal and consumer electronics using different approaches including experimental, analytical, numerical and hybrid; i.e. a combination of two or more methods [4−8]. These approaches investigate the thermal behaviour of solid and porous fins under different operating conditions. Examples of some related research includes Runge-Kutta [9−11], Galerkin's method of weighted residual [12,13], least square method [14], various collocation methods - Haar wavelet [15,16], - spectral [17], - Chebyshev [18,19]; Spectral element [20]; and Legendre [21] Adomian decomposition method [22,23], Differential transform method [24−26], variational iteration method [27], and Homotopy analysis method [28], hybrid methods [29−31]. Moreover, different methods of heat transfer enhancement in thermal and electronic systems have been established in the literature with the heat sink as a critical component to control heat dissipation to an operational level [32−36].

Nevertheless, to achieve the nonlinear analysis of the heat transfer problem, it is often a daunting task to develop the generalized closed-form solution. Approximate analytical methods are useful in developing the nonlinear closed-form solution of the heat transfer problem but not without some constraints. One key limitation of the approximate analytical methods is the generation of large expressions with many complex terms. Consequently, recourse is often made to numerical methods for the nonlinear analysis of the heat problem. Among the numerical methods, PSCM presents an efficient approach to obtain solutions to nonlinearity issue even with the complexities of boundary conditions. PSCM uses Lagrange interpolation polynomials for the approximations, and Chebyshev-Gauss-Lobatto (CGL) points for the orthogonal collocation or spatial discretization. PSCM is an efficient numerical method that transforms differential and integral expressions into some algebraic equations which are easily solved numerically. PSCM is generally easy to implement and yields very accurate results and gives satisfactory approximations over most existing numerical methods. Therefore, the high level of accuracy achieved using PSCM motivates its applications in the study of thermal performance of heat transfer surfaces.

In the present work, we investigate the effect of inclination on the thermal performance of a porous fin heat sink. To the best of the author's knowledge, the study on the effect of inclination on the performance of a single fin; either solid or porous in heat sinks has not been carried out in the literature. To this end, PSCM is applied numerically to solve the effect of inclination and heat generation on the thermal behaviour of a porous fin heatsink. The study aims at thermal characterization of heat sink with inclined porous fins of rectangular geometry. The physical model of the formulation is presented in Section 2. In Section 3 the developed nonlinear heat transfer equation is solved using PSCM, and the obtained results are discussed in Section 4. Section 5 summarises the various findings from the analysis.

2. Problem formulation

Fig. 1 shows a heatsink of vertical fins. The geometry of the fin is of length L, thickness t and is exposed on both faces to a convective-radiative environment at temperature T_∞. To simplify the formulation of fin problem, we made the following assumptions [26,37]:

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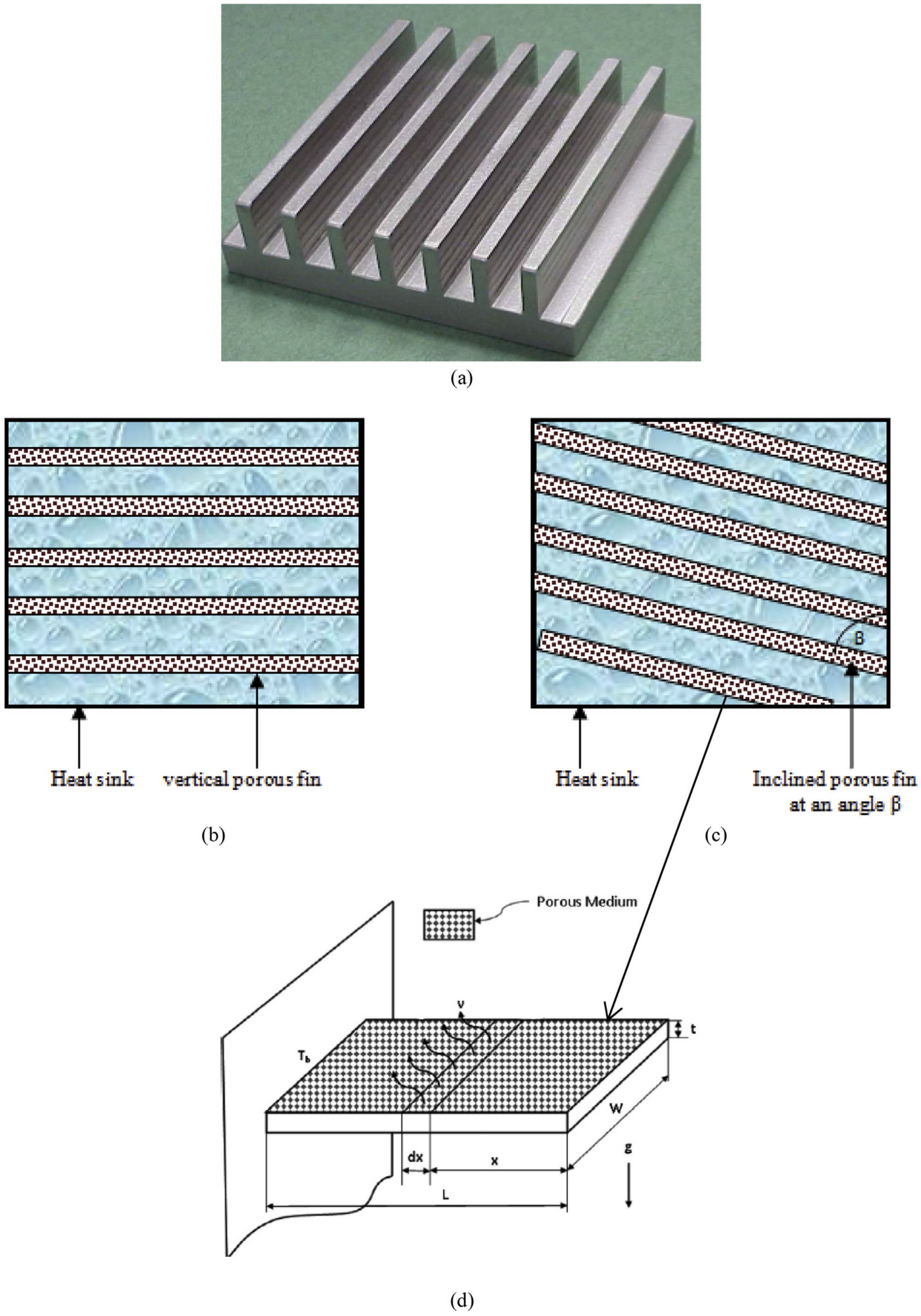

Fig. 1. (**a**) Porous heat sink. (**b**) Plain view of a vertical fins heat sink. (**c**) Plain view of the inclined porous fins heat sink. (d) Porous fin with geometrical parameter.

1. Porous media is homogeneous and saturated with single-phase fluid.
2. The interaction between the saturated fluid and medium is governed by Darcy's model.
3. Physical properties of the porous fin and the fluid are constant.
4. The fin thickness is small compared with its width and length, so that temperature gradients across the

fin thickness and heat transfer from the edges of the fin may be neglected.

5. Fin tip is adiabatic.

The steady-state one-dimensional thermal model of a single porous fin heat sink as established from our previous works is in the form [12,18,37]:

$$\frac{d}{dx}\left[\left(1+\psi(T-T_a)\right)\frac{dT}{dx}\right]-\frac{\rho c_p g K\beta_{th}}{k_{eff}tv}(T-T_a)^2 - \frac{h_{eff}}{k_{eff}t}(T-T_a)+\frac{q(T)}{k_{eff}A_{cr}}=0 \tag{1}$$

The boundary conditions are

$$x=0,\quad \frac{dT}{dx}=0 \qquad x=L,\quad T=T_b \tag{2}$$

where the internal heat generation is expressed as:

$$q(T)=q_0[1+\lambda(T-T_\infty)] \tag{3}$$

If we substitute Equation (3) into Equation (1), we arrive at

$$\frac{d}{dx}\left[\left(1+\psi(T-T_a)\right)\frac{dT}{dx}\right]-\frac{\rho c_p g K\beta_{th}}{k_{eff}tv}(T-T_a)^2 - \frac{h_{eff}}{k_{eff}t}(T-T_a)+\frac{q_o}{k_{eff}A_{cr}}[1+\lambda(T-T_\infty)]=0 \tag{4}$$

The effective coefficient of heat transfer ($h_{eff} = (kNu)/L$) is found from the correlations [42]

$$Nu=\begin{cases}0.68+\dfrac{0.67[(Grcos\beta)Pr]^{0.25}}{\left\{1+\left(\dfrac{0.492}{Pr}\right)^{0.5625}\right\}^{0.444}} & (Grcos\beta)Pr<10^4\\ 0.59[(Grcos\beta)Pr]^{0.25} & 10^4<(Grcos\beta)Pr<10^9\end{cases} \tag{5}$$

alternatively, for all values of $(Grcos\beta)Pr$

$$Nu=\left\{0.825+\frac{0.387[(Grcos\beta)Pr]^{0.167}}{\left\{1+\left(\frac{0.492}{Pr}\right)^{0.5625}\right\}^{0.296}}\right\}^2 \tag{6}$$

where the inclination angle is expressed as “β” which is “0” for the vertical or uninclined fin.

By introducing the dimensionless parameters of Equation (7) in Equation (4)

$$X=\frac{x}{L},\theta=\frac{T-T_a}{T_b-T_a},S_h=\frac{\rho c_p g k\beta_{th}L^2(T_b-T_a)}{tvk_{eff}},$$
$$M^2=\frac{h_{eff}L^2}{k_{eff}t},Q=\frac{q_oL^2t}{k_{eff}A_{eeff}(T_b-T_a)},\gamma=\lambda(T_b-T_a),$$
$$\zeta=\psi(T_b-T_a) \tag{7}$$

we arrive at

$$\frac{d^2\theta}{dX^2}+\zeta\theta\frac{d^2\theta}{dX^2}+\zeta\left(\frac{d\theta}{dX}\right)^2-S_h\theta^2-M^2\theta+M^2Q(1+\gamma\theta)=0 \tag{8}$$

Therefore, Equation (8) becomes the nonlinear dimensionless thermal model, and the dimensionless boundary condition becomes

$$X=0,\quad \frac{d\theta}{dX}=0 \qquad X=1,\quad \theta=1 \tag{9}$$

3. Application of pseudospectral collocation method to the heat transfer problem

Equation (8) is a non-linear ordinary differential equation which is to be solved alongside with boundary conditions in Eq. (9). In order to solve the nonlinear equations of Eq. (8), pseudospectral collocation method is applied. Therefore, rearranging Eq. (8) as:

$$\left\{\left(1+\zeta\theta^*\right)\frac{d^2}{dX^2}-M^2+M^2Q\gamma\right\}\theta^* = S_h\left(\theta^*\right)^2-\zeta\left(\frac{d\theta^*}{dX}\right)^2-M^2Q \tag{10}$$

where θ^* is the value of the last iteration of the dimensionless temperature.

The application of pseudospectral collocation method requires that the spatial discretization of the dimensionless thermal model should be carried out through the utilization of Chebyshev-Gauss-Lobatto collocation points.

$$\Psi_i = cos\left[\frac{\pi(i-1)}{N-1}\right],\ \mathrm{i} = 1,\ 2,\&,\mathrm{N} \tag{11}$$

The collocation points are mapped out in the computation domain of [-*1, 1*]. In order to transform any arbitrary (*X, [0, 1*]) interval into standard interval (*Ψ*, [-*1, 1*]), using a transformation using algebraic mapping *X* = *(Ψ+1)/2.*

Following the principle of pseudospectral collocation method, with the aid of Lagrange interpolation shown in Eq. (12), the unknown dimensionless temperature is approximated to dimensionless temperature points on the collocation points

$$\sum_{i=1}^{N} \Theta(\Psi_i)\hbar_i(\Psi), \tag{12}$$

where $\hbar_i$ is the Lagrange interpolation polynomial given by

$$\hbar_i(\Psi) = \frac{\frac{w_i}{(\Psi-\Psi_i)}}{\sum_{j=1}^{N}\frac{w_j}{(\Psi-\Psi_j)}} \tag{13}$$

and

$$w_i = (-1)^{i-1}\delta_j\ \delta_j = \begin{cases} 1/2 & j = 1,N \\ 1 & i = 2,3,\ldots,N-1 \end{cases}$$

substituting Equation (12) in Equation (10) and also into the boundary conditions in Eq. (9) to obtain the spectral discretized algebraic equations which are written in matrix form as:

$$\mathbf{A}\Theta = \mathbf{B},\quad i = 1,2,\ldots,N \tag{14}$$

and the boundary conditions are

$$D_{1j}\Theta = 0,\quad \sum_{j=1}^{N}\theta_J = 1 \tag{15}$$

The element expressions for matrix ***A*** and ***B*** are

$$\mathbf{A}_{ij} = \begin{cases} (1+\zeta\Theta^*)D_{ij}^{(2)} - M^2 + M^2Q\gamma & i = j \\ (1+\zeta\Theta^*)D_{ij}^{(2)} & i \neq j \end{cases} \tag{16}$$

$$\mathbf{B}_i = S_h\left[\sum_{j=1}^{N}\Theta_j^*\right]^2 - \zeta\left[\sum_{j=1}^{N}D_{ij}^{(1)}\Theta_j^*\right] - M^2Q \tag{17}$$

It is worth noting that the entries of the coefficient matrices of the 1st and 2nd order derivative are represented by $D_{ij}^{(1)}$ and $D_{ij}^{(2)}$, respectively.

Using the algorithm outlined below, we implement the pseudospectral collocation method by:

Step 1 Input the number of collocation points, and compute the coordinate values of the nodes for the matrices of the 1st and 2nd order derivative.
Step 2 Set the dimensionless temperature at initial values in all directions except at the boundaries.
Step 3 Use Eqs. (16) and (17), assemble the matrices A and B.
Step 4 Imposed the boundary conditions in Eq. (15) in the algebraic systems of equations and solve the equation directly.
Step 5 Using the convergence criteria $\sum|\phi_i^p - \phi^{p-1}| \le 10^{-12}$, terminate the iteration procedure,

Else.

Step 6 Return to Step 3.

4. Result and discussion

Table 1 highlights the comparison of the result of PSCM and an established Runge-Kutta result. From

Table 1
Comparison of temperature results.

x	Numerical Method (Runge-Kutta)	PSCM (Present Study)	Absolute error
0.00	0.863499231	0.863499242	0.000000011
0.05	0.863828568	0.863828561	0.000000007
0.10	0.864817090	0.864817085	0.000000005
0.15	0.866466182	0.866466173	0.000000009
0.20	0.868776709	0.868776717	0.000000008
0.25	0.871751555	0.871751544	0.000000011
0.30	0.875393859	0.875393871	0.000000012
0.35	0.879707472	0.879707058	0.000000014
0.40	0.884696967	0.884696909	0.000000006
0.45	0.890367650	0.890367657	0.000000007
0.50	0.896725569	0.896725559	0.000000010
0.55	0.903777531	0.903777545	0.000000014
0.60	0.911531120	0.911531105	0.000000015
0.65	0.919994710	0.919994727	0.000000017
0.70	0.929177488	0.929177477	0.000000011
0.75	0.939089476	0.939089458	0.000000018
0.80	0.949741555	0.949741567	0.000000012
0.85	0.961145491	0.961145471	0.000000020
0.90	0.973313964	0.973313954	0.000000010
0.95	0.986260599	0.986260591	0.000000008
1.00	1.000000000	1.000000000	0.000000000

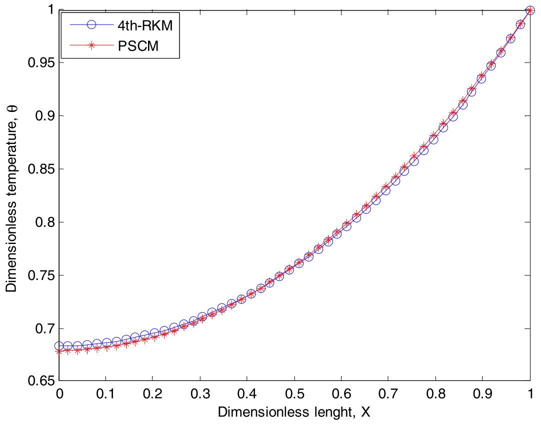

Fig. 2. Comparison of results of Fourth-order Runge-Kutta and pseudo-spectral collocation method.

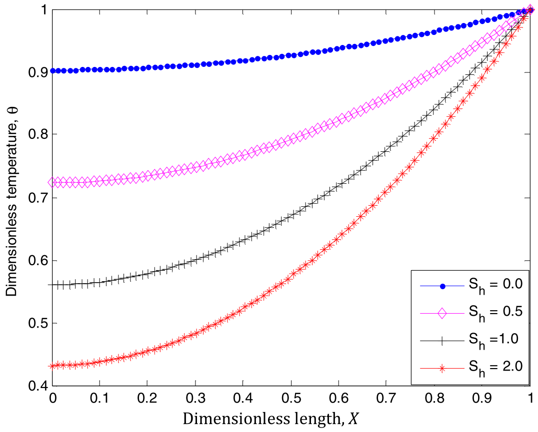

Fig. 4. Porosity effect on the performance of the inclined fin.

Table 1, the result of PSCM and Runge-Kutta agree excellently, which validates its accuracy for nonlinear heat problem.

Fig. 2 shows the comparison of results of Fourth-order Runge-Kutta and pseudo-spectral collocation method. From the figure, it is established that the results of PSCM and the fourth-order Runge-Kutta agree excellently. This verifies the accuracy of PSCM in providing a reliable numerical solution to the nonlinear problem.

Based on the algorithm given in the preceding section, the simulated results are presented in Figs. 3—8. Fig. 3 highlights the inclination effect on the efficiency of the porous fin. It can be seen from Fig. 3 that the inclined porous fin exhibits improved thermal performance than the corresponding vertical heat sink of equal dimension. Moreover, the thermal performance of the inclined or tilted fin increases with decrease in the length-thickness aspect ratio of the fin. Furthermore, Fig. 3 shows that the increasing angle inclination of the porous fin enhances the heat transfer performance of the fin. This enhancement in the inclined fin is likely as a result of increased velocity and reduced wake flow region behind the fin.

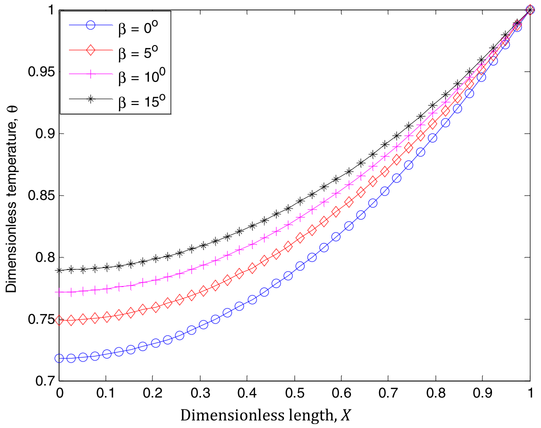

Fig. 3. Inclination effect on the fin performance.

Fig. 4 shows the effect of porosity on the temperature distribution of the fin whilst Fig. 5a and b illustrate the effects temperature-dependent thermal conductivity parameter on the performance of the inclined fin. From Fig. 4, it can be observed that the temperature of the porous fin decreases and drops rapidly as the porosity parameter increases. This overall implication of this effect is that increase in porosity improves the performance of the fin.

Fig. 6 shows the effect of heat generated internally on the temperature distribution of porous fin, whilst Fig. 7 presents the influence of the temperature-dependent internal heat generation on the temperature distribution of the fin. From Figs. 6 and 7, it can be observed that the temperature gradient of the fins decreases, as the internal heat generation parameters increases which subsequently decreases the rate of heat transfer of the fin.

Figs. 8 and 9 highlight the effect of thermo-geometric and porosity variable on the efficiency ratio of the inclined fin to vertical fin. From these figures, it can be observed that the efficiency ratio of the inclined fin to the vertical fin is greater than one for all considered cases. Moreover, the increased temperature distribution by convection in the inclined porous fin is due to the effective shorter length of the fin caused by the tilted angle, β as depicted in Fig. 8.

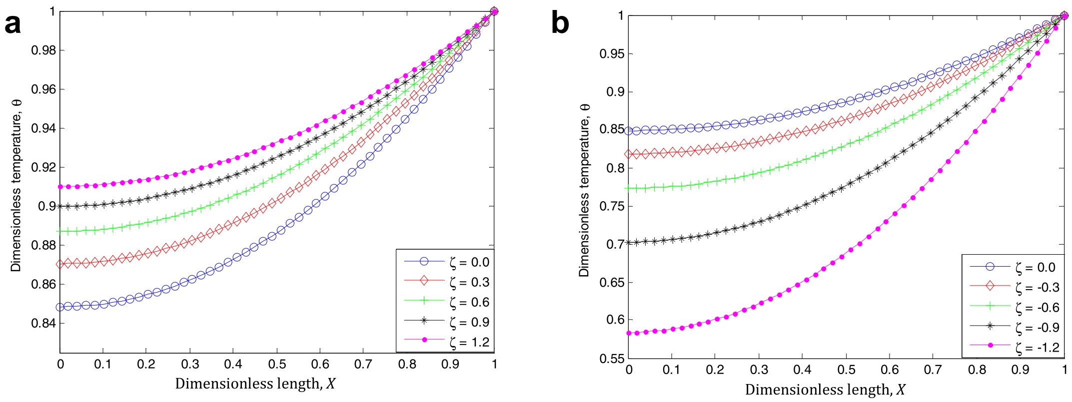

Fig. 5. Effect of thermal conductivity parameter on the inclined fin performance.

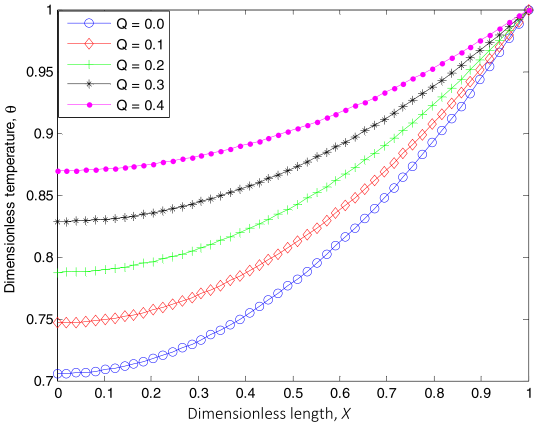

Fig. 6. Effect of heat generation on performance of the inclined fin.

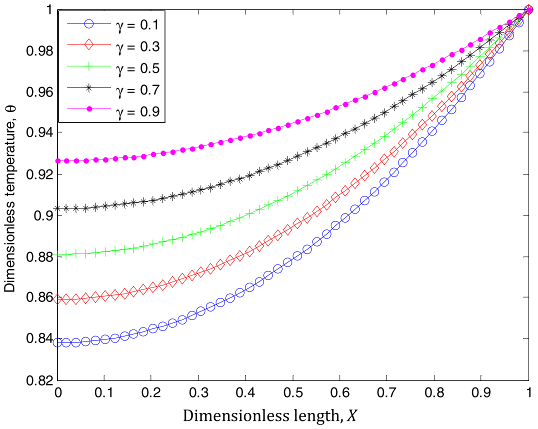

Fig. 7. Effect of temperature-dependent heat generation on the performance of the inclined fin.

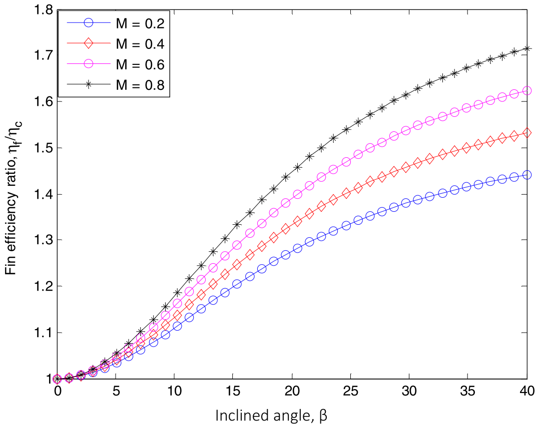

Fig. 8. Thermo-geometric variable effect on efficiency ratio of inclined to vertical porous fins.

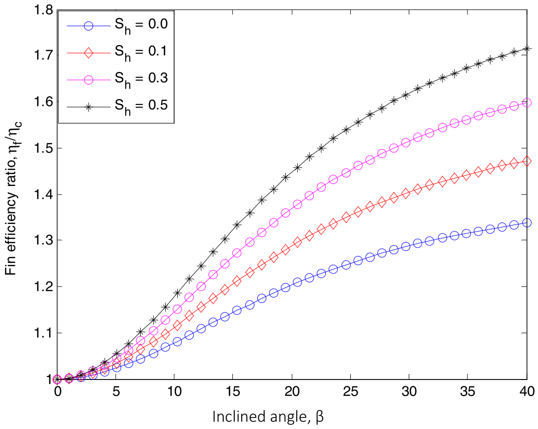

Fig. 9. Porosity variable effect on efficiency ratio of inclined to vertical porous fins.

5. Conclusion

In this study, the effect of inclination effect on the thermal performance of a porous fin heatsink has been investigated using pseudospectral collocation method. From the analysis, based on the parameter used for the numerical investigation, it is established that the inclined porous fin in heat sink shows improved thermal performance than the corresponding vertical heat sink of the same size. In addition, the performance of the inclined or tilted fin increases with decrease in the length-thickness aspect ratio of the fin. Furthermore, the study reveals that as the internal heat generation parameter increases the dimensionless temperature profile of the porous fin increases, which consequently, increases the rate of heat transfer in the fin. The obtained results using pseudospectral collocation method highlights the reliability of the method for analysis of nonlinear heat transfer problems.

Nomenclature

A	Fin cross-sectional area
A_b	Base area of the fin
A_s	Fin surface area
h_{eff}	Heat coefficient at fin base
K	Permeability
M	Thermo-geometric parameter
$\dot{m}$	Saturated fluid mass flowage
Nu	Nusselt number
P	Fin perimeter
t	Fin thickness
X	dimensionless length
q	Internal heat generation

Greek Symbols

β	Inclination angle
θ	Temperature (Dimensionless)
η	Fin efficiency
β_{th}	Coefficient of thermal expansion
υ	Kinematic viscosity
ρ	Fluid density

References

[1] S. Kiwan, M.A. Al-Nimr, Using porous fins for heat transfer enhancement, J. Heat Tran. 123 (2000) 790−795.

[2] S.E. Ghasemi, M. Hatami, D.D. Ganji, Thermal analysis of convective fin with temperature-dependent thermal conductivity and heat generation, Case Stud. Therm. Eng. 4 (2014) 1−8, 11/01/2014.

[3] R. Das, Estimation of parameters in a fin with temperature-dependent thermal conductivity and radiation, Proc. IME E J. Process Mech. Eng. 230 (2016) 474−485.

[4] Z. Li, M. Sheikholeslami, M. Jafaryar, A. Shafee, A.J. Chamkha, Investigation of nanofluid entropy generation in a heat exchanger with helical twisted tapes, J. Mol. Liq. 266 (2018) 797−805.

[5] G.A. Oguntala, G. Sobamowo, N.N. Eya, R.A. Abd-Alhameed, Investigation of simultaneous effects of surface roughness, porosity and magnetic field of rough porous micro-fin under a convective-radiative heat transfer for improved microprocessor cooling of consumer electronics, IEEE Trans. Compon. Packag. Manuf. Technol. (2018), 1−11 (in press).

[6] F. Selimefendigil, H.F. Öztop, Fuzzy-based estimation of mixed convection heat transfer in a square cavity in the presence of an adiabatic inclined fin, Int. Commun. Heat Mass Tran. 39 (2012) 1639−1646.

[7] F. Selimefendigil, F. Bayrak, H.F. Oztop, Experimental analysis and dynamic modeling of a photovoltaic module with porous fins, Renew. Energy 125 (2018) 193−205.

[8] F. Selimefendigil, H.F. Öztop, Effects of an adiabatic fin on the mixed convection heat transfer in a square cavity with two ventilation ports, Therm. Sci. 18 (2014) 377−389.

[9] S. Kiwan, O. Zeitoun, Natural convection in a horizontal cylindrical annulus using porous fins, Int. J. Numer. Methods Heat Fluid Flow 18 (2008) 618−634.

[10] R.S.R. Gorla, A.Y. Bakier, Thermal analysis of natural convection and radiation in porous fins, Int. Commun. Heat Mass Tran. 38 (2011) 638−645.

[11] S. Saedodin, S. Sadeghi, Temperature distribution in long porous fins in natural convection condition, Middle East J. Sci. Res. 13 (2013) 812.

[12] M.G. Sobamowo, O.M. Kamiyo, O.A. Adeleye, Thermal performance analysis of a natural convection porous fin with temperature-dependent thermal conductivity and internal heat generation, Therm. Sci. Eng. Prog. 1 (2017) 39−52.

[13] B. Alshuraiaan, K. Khanafer, The effect of the position of the heated thin porous fin on the laminar natural convection heat transfer in a differentially heated cavity, Int. Commun. Heat Mass Tran. 78 (2016) 190−199.

[14] A.R. Shateri, B. Salahshour, Comprehensive thermal performance of convection−radiation longitudinal porous fins with various profiles and multiple nonlinearities, Int. J. Mech. Sci. 136 (2018) 252−263.

[15] G.A. Oguntala, R.A. Abd-Alhameed, Haar wavelet collocation method for thermal analysis of porous fin with temperature-dependent thermal conductivity and internal heat generation, J. Appl. Comput. Mech. 3 (2017) 185−191.

[16] G. Oguntala, R. Abd-Alhameed, G. Sobamowo, I. Danjuma, Performance, thermal stability and optimum design analyses of rectangular fin with temperature-dependent thermal properties and internal heat generation, J. Comput. Appl. Mech. 49 (2018) 37−43.

[17] J. Ma, Y. Sun, B. Li, H. Chen, Spectral collocation method for radiative−conductive porous fin with temperature dependent properties, Energy Convers. Manag. 111 (2016) 279−288.

[18] G.A. Oguntala, R.A. Abd-Alhameed, G.M. Sobamowo, N. Eya, Effects of particles deposition on thermal performance of a convective-radiative heat sink porous fin of an electronic component, Therm. Sci. Eng. Prog. 6 (2018) 177−185.

[19] G. Oguntala, R. Abd-Alhameed, Thermal analysis of convective-radiative fin with temperature-dependent thermal

conductivity using chebychev spectral collocation method, J. Appl. Comput. Mech. 4 (2018) 87−94, 2018.

[20] J.S. Ma, Yasong, Benwen Li, Simulation of combined conductive, convective and radiative heat transfer in moving irregular porous fins by spectral element method, Int. J. Therm. Sci. 118 (2017) 475−487.

[21] L. Jayesimi, G. Oguntala, Prediction of temperature distribution in straight fin with variable thermal conductivity and internal heat generation using Legendre wavelet collocation method, J. Comput. Appl. Mech. 48 (2017) 217−224.

[22] P.K. Roy, H. Mondal, A. Mallick, A decomposition method for convective−radiative fin with heat generation, Ain Shams Eng. J. 6 (2015) 307−313.

[23] D. Bhanja, B. Kundu, P.K. Mandal, Thermal analysis of porous pin fin used for electronic cooling, Procedia Eng. 64 (2013) 956−965.

[24] A. Moradi, T. Hayat, A. Alsaedi, Convection-radiation thermal analysis of triangular porous fins with temperature-dependent thermal conductivity by DTM, Energy Convers. Manag. 77 (2014) 70−77.

[25] M. Torabi, H. Yaghoobi, Series solution for convective-radiative porous fin using differential transformation method, J. Porous Media 16 (2013) 341−349.

[26] G. Oguntala, R. Abd-Alhameed, Performance of convective-radiative porous fin heat sink under the influence of particle deposition and adhesion for thermal enhancement of electronic components, Karbala Int. J. Mod. Sci. 4 (3) (2018) 297−312.

[27] S.B. Coşkun, M.T. Atay, Fin efficiency analysis of convective straight fins with temperature dependent thermal conductivity using variational iteration method, Appl. Therm. Eng. 28 (2008) 2345−2352.

[28] H. Hoshyar, D.D. Ganji, M. Abbasi, Determination of temperature distribution for porous fin with temperature-dependent heat generation by homotopy analysis method, J. Appl. Mech. Eng. 4 (2015) 153.

[29] S. Mosayebidorcheh, M. Sheikholeslami, M. Hatami, D.D. Ganji, Analysis of turbulent MHD Couette nanofluid flow and heat transfer using hybrid DTM−FDM, Particuology 26 (2016) 95−101.

[30] S. Rezazadeh Amirkolaei, D.D. Ganji, H. Salarian, Determination of temperature distribution for porous fin which is exposed to uniform magnetic field to a vertical isothermal surface by homotopy analysis method and collocation method, Indian J. Sci. Res. 1 (2014) 215−222.

[31] H. Hoshyar, I. Rahimipetroudi, D.D. Ganji, A.R. Majidian, Thermal performance of porous fins with temperature-dependent heat generation via the homotopy perturbation method and Collocation method, J. Appl. Math. Comput. Mech. 14 (2015) 53−65.

[32] M. Sheikholeslami, M. Jafaryar, D.D. Ganji, Z. Li, Exergy loss analysis for nanofluid forced convection heat transfer in a pipe with modified turbulators, J. Mol. Liq. 262 (2018) 104−110.

[33] M. Sheikholeslami, M. Jafaryar, S. Saleem, Z. Li, A. Shafee, Y. Jiang, Nanofluid heat transfer augmentation and exergy loss inside a pipe equipped with innovative turbulators, Int. J. Heat Mass Tran. 126 (2018) 156−163, 11/01/2018.

[34] M. Sheikholeslami, M. Jafaryar, A. Shafee, Z. Li, Nanofluid heat transfer and entropy generation through a heat exchanger considering a new turbulator and CuO nanoparticles, J. Therm. Anal. Calorim. 134 (2018) 2295−2303.

[35] F. Selimefendigil, H.F. Öztop, Numerical analysis of laminar pulsating flow at a backward facing step with an upper wall mounted adiabatic thin fin, Comput. Fluids 88 (2013) 93−107.

[36] M. Alam, S. Hossain, M. Simanto, M. Mithu, M. Islam, Experimental and Numerical Investigation of an Air to Water Heat Exchanger " Presented at the Nternational Conference on Mechanical Engineering and, Renewable Energy Chittagong University of Engineering & Technology, Bangladesh, 2015.

[37] G. Oguntala, R. Abd-Alhameed, G. Sobamowo, On the effect of magnetic field on thermal performance of convective-radiative fin with temperature-dependent thermal conductivity, Karbala Int. J. Mod. Sci. 4 (2018) 1−11.