Analysis of Dynamic Behaviour of a Tensioned Carbon Nanotube in Thermal and Pressurized Environments

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Abstract

In this paper, the dynamic behaviour of a tensioned single-walled carbon nanotubes (SWCNT) in thermal and pressurized environments is investigated analytically. With the applications of Bernoulli-Euler and thermal elasticity mechanics theories, the governing equation of motion are developed and solved using Laplace and Fourier transforms. The results of the close form solution in this work are in excellent agreements with past results in literature. From the parametric studies, it is established that as the magnitude of the pressure distribution at the surface increases, the deflection associated with the nanotube increases at any mode of vibration. However, a corresponding increase in the temperature and foundation parameter have an attenuating attribute on deflection. Also, it is shown that the frequency of the vibration increases as the Winkler parameter increases and the mass of the SWCNT decreases. It is envisaged that this work will enhance the use of SWCNT in structural, electrical and mechanical applications.

Keywords

Thermal and Pressurized environments, Single-walled carbon nanotube, Exact analytical solution, Integral transform, Dynamic analysis

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1. Introduction

The discovery of carbon nanotube by Iijima [1] has led to various logical investigations and analyses of carbon nanotube. This is because of the excellent thermal, electrical, structural and mechanical properties of the structure. In order to explore the behaviours of the carbon nanotube under different conditions, Torrones et al. [2] analyzed straight SWCNTs with emphasis on the molecular junctions while Nagy et al. [3] presented a study on Y-branched SWCNTs. In the last few decades, some studies [4–7] have been presented in order to investigate the effects of multiple walls on the dynamic behaviour and deformation of carbon nanotube. Also, the material properties, mechanics and stress analysis of CNTs have been explored in some past research works [8–12]. Furthermore, in recent studies, the theories of Euler-Bernoulli and Timoshenko have been applied to model and study the dynamic behaviours of SWCNTs and MWCNTs [13–20]. The impacts of piezoelectric and electrostatic actuators on the vibration characteristics of carbon nanotube were examined by Quakad and Younis [21], and Zamanian et al. [22]. In some further studies, Abdel-Rahman and Nayfeh [23], Hawwa and Al-Qahtani [24], Hajnayeb and Khaden [25] and Belhadj et al. [26] presented different works on the vibration and instability of DWCNTs under the influence of electrostatic actuations. In the later work, a DWCNT was situated and conditioned to a direct and alternating voltage and different behaviours of the nanotubes were recorded as the exciting agent was varied. Also, the bifurcation point of the DWCNT was determined. It was concluded that both walls have the same frequency of vibration under the two resonant conditions considered. In another work, Belhadj et al. [26] examined the vibration behaviour of a simply supported SWCNT employing the nonlocal theory of elasticity. The authors obtained the natural frequency up to third mode while they also established high level of frequency. Such frequency may be harnessed to be of high merit in optical applications. Lei et al. [27] studied the dynamic behaviour of DWCNT by employing the well-known Timoshenko theory of beam. Asgharifard and Haeri [28] derived the nonlinear governing equations of vibration and relations which are applied to nanobeams that are functionally graded and have surface roughness. Wang [29] developed a close form solution for a

functionally graded nanobeam under the influence of surface roughness. The significance of the study was ascertained for reasonably small thickness of the nanotube. In some other studies [30–35], effects of elastic foundations (Winkler, Pasternak and Visco-Pasternak medium) on the dynamic response of nanotubes have been investigated. Other interesting works through modelling and experiment have also been presented to justify the widespread application of SWCNTs [36–40]. Furthermore, Sobamowo and Yinusa [41] applied homotopy perturbation method to study the dynamic response of a SWCNT under the influences of thermofluidic properties. Sheikholeslami [42] analyzed nanofluid forced convection in a porous cubic cavity with the consideration of Lorentz force. The author concluded that when parameters such as Darcy number, Reynolds number and nano-particle fraction are augmented, the average Nusselt number is noticeably enhanced. In another work, Sheikholeslami et al. [43] presented a further work on the effects of Lorentz force on heat transfer during solidification process. Ghadikolaei et al. [44] investigated the squeezing flow of ethylene glycol ($C_2H_6O_2$) carbon nanotubes (CNTs) in rotating stretching channel with nonlinear thermal radiation. Jamshidi et al. [45], put forward an interesting study on the application of energy balance method and variational iteration method to an oscillation of a mass attached to a stretched elastic wire. However, in such nonlinear dynamic analyses, it is highly required to develop closed form solutions to set benchmarks for the numerical and approximate analytical solutions developed for the nonlinear models. Integral transforms provide direct ways of describing the general close form solution of a problem when inverted. This is because the methods map the governing equations into another domain in algebraic forms while preserving all the parameters. Motivated by these considerations, this work applies integral transforms to study the dynamic response of a SWCNT in a thermal and pressurized environment. Also, parametric studies are carried out to establish the effects of the model parameters on the vibration characteristics of the nanotube.

2. Problem description and governing equation

Consider a SWCNT with a uniformly distributed surface pressure as shown in Fig. 1. In order to develop

Fig. 1. Schematic of the SWCNT with uniformly distributed pressure at the surface.

the equation of motion, the following assumptions are made:

- (i) the SWCNT is modeled according to Euler-Bernoulli beam
- (ii) the exciting agent is a uniform pressure distribution at the SWCNT surface governed by:

$$P(x) = \mu A^{CNT} \frac{d}{dx} \left(P_0 \left(1 + \frac{\delta}{L^{CNT}} x \right) \right). \quad (1)$$

- (iii) the SWCNT is simply supported at both ends.
- (iv) CNT is homogenous and has constant thermal, mechanical and physical properties.
- (v) CNT has a constant cross-section, hence there is a constant moment of Inertia.

By incorporating the above assumptions into the classic Euler-Bernoulli beam model, the vibration of Fig. 1 can be described by the model:

$$EI^{CNT} \frac{\partial^4 \chi}{\partial x^4} + \left(\frac{EA}{1-2\nu^*} \alpha^* \theta - T \right) \frac{\partial^2 \chi}{\partial x^2} + M \frac{\partial^2 \chi}{\partial t^2} + K \chi = P(x). \quad (2)$$

If one substitutes Eq. (1) into Eq. (2) and simplify the resulting equation, the model becomes;

$$EI^{CNT} \frac{\partial^4 \chi}{\partial x^4} + \left(\frac{EA}{1-2\nu^*} \alpha^* \theta - T \right) \frac{\partial^2 \chi}{\partial x^2} + M \frac{\partial^2 \chi}{\partial t^2} + K \chi = \mu A^{CNT} \frac{d}{dx} \left(P_0 \left(1 + \frac{\delta}{L^{CNT}} x \right) \right). \quad (3)$$

3. Methods of solution using integral transforms

In order to develop a closed-form solution to the governing equation, Laplace and Fourier Transforms are used. The dynamic governing equation as shown in Eq. (3) may be expressed as,

$$M \frac{\partial^2 \chi}{\partial t^2} + EI^{CNT} \frac{\partial^4 \chi}{\partial x^4} + \left(\frac{EA}{1-2\nu^*} \alpha^* \theta - T \right) \frac{\partial^2 \chi}{\partial x^2} + K \chi = \mu A^{CNT} P_0 \frac{\delta}{L^{CNT}} \quad (4)$$

The initial and boundary conditions are given as

$$\begin{aligned} \chi(x, 0) = \dot{\chi}(x, 0) = 0, \quad \chi(0, t) = \chi''(0, t) \\ = 0, \quad \chi(L^{CNT}, t) = \chi''(L^{CNT}, t) = 0 \end{aligned}$$

Applying Laplace transform on the temporal term, gives

$$\begin{aligned} M[s^2 \bar{\chi}(x, s) - \dot{\chi}(x, 0) - s\chi(x, 0)] + EI^{CNT} \frac{d^4 \bar{\chi}}{dx^4} \\ + \left(\frac{EA}{1-2\nu^*} \alpha^* \theta - T \right) \frac{d^2 \bar{\chi}}{dx^2} + K \bar{\chi} = \mu A^{CNT} P_0 \frac{\delta}{s L^{CNT}}, \end{aligned} \quad (5)$$

using the initial conditions and applying Fourier transform to the spatial terms, we have

$$\begin{aligned} M s^2 \tilde{\tilde{\chi}} + EI^{CNT} \left[\left(\frac{i\pi}{L^{CNT}} \right)^4 \tilde{\tilde{\chi}} - \left(\frac{i\pi}{L^{CNT}} \right)^3 [\chi(0) - (-1)^i \chi(L^{CNT})] + \left(\frac{i\pi}{L^{CNT}} \right) [\chi''(0) - (-1)^i \chi''(L^{CNT})] \right] + \\ \left(\frac{EA}{1-2\nu^*} \alpha^* \theta - T \right) \left[- \left(\frac{i\pi}{L^{CNT}} \right)^2 \tilde{\tilde{\chi}} + \left(\frac{i\pi}{L^{CNT}} \right) [\chi(0) - (-1)^i \chi(L^{CNT})] \right] + K \tilde{\tilde{\chi}} = \mu A^{CNT} P_0 \frac{\delta}{s L^{CNT}} \left(\frac{L^{CNT}}{i\pi} \right) [1 - (-1)^i]. \end{aligned} \quad (6)$$

Following the application of the boundary conditions and grouping the like terms, one arrives at

$$\tilde{\chi} = \frac{\mu A^{CNT} P_0 \delta\left(\frac{1}{i\pi}\right) [1 - (-1)^i]}{s \left(Ms^2 + EI^{CNT} \left(\frac{i\pi}{L^{CNT}}\right)^4 - \left(\frac{EA}{1-2\nu^*} \alpha^* \theta - T\right) \left(\frac{i\pi}{L^{CNT}}\right)^2 + K \right)} \quad (7)$$

3.1. Determination of the natural frequency

$$\tilde{\chi} = \frac{\mu A^{CNT} P_0 \delta\left(\frac{1}{i\pi}\right) [1 - (-1)^i]}{s \left\{ \left(s - j \left\{ \frac{1}{M} \left(EI^{CNT} \left(\frac{i\pi}{L^{CNT}}\right)^4 - \left(\frac{EA}{1-2\nu^*} \alpha^* \theta - T\right) \left(\frac{i\pi}{L^{CNT}}\right)^2 + K \right) \right\} \right) \cdot \left(s + j \left\{ \frac{1}{M} \left(EI^{CNT} \left(\frac{i\pi}{L^{CNT}}\right)^4 - \left(\frac{EA}{1-2\nu^*} \alpha^* \theta - T\right) \left(\frac{i\pi}{L^{CNT}}\right)^2 + K \right) \right\} \right) \right\}} \quad (8)$$

The natural frequencies can be determined from the poles of Eq. (8)

$$\omega_i = \sqrt{\left\{ \frac{1}{M} \left(EI^{CNT} \left(\frac{i\pi}{L^{CNT}}\right)^4 - \left(\frac{EA}{1-2\nu^*} \alpha^* \theta - T\right) \left(\frac{i\pi}{L^{CNT}}\right)^2 + K \right) \right\}} \quad (9)$$

Applying the inverse Laplace Transform of Eq. (8), yields

$$\tilde{\chi} = \frac{\mu A^{CNT} P_0 \delta(L^{CNT})^4 \left[1 + (-1)^{i+1} + \cosh \left(\frac{\sqrt{(L^{CNT})^4 M \left(-EI^{CNT} (i\pi)^4 + \left(\frac{EA}{1-2\nu^*} \alpha^* \theta - T\right) (i\pi L^{CNT})^2 - K (L^{CNT})^4 \right)}}{(L^{CNT})^4 M} t \right) (1 + (-1)^i) \right]}{i\pi \left(EI^{CNT} (i\pi)^4 - \left(\frac{EA}{1-2\nu^*} \alpha^* \theta - T\right) (i\pi L^{CNT})^2 + K (L^{CNT})^4 \right)} \quad (10)$$

and the Fourier inverse of Eq. (10) gives

$$\chi(x, t) = \frac{2}{L^{CNT}} \left\{ \sum_{i=1}^{\infty} \frac{\mu A^{CNT} P_0 \delta(L^{CNT})^4 \left[1 + (-1)^{i+1} + \cosh \left(\frac{\sqrt{(L^{CNT})^4 M \left(-EI^{CNT} (i\pi)^4 + \left(\frac{EA}{1-2\nu^*} \alpha^* \theta - T\right) (i\pi L^{CNT})^2 - K (L^{CNT})^4 \right)}}{(L^{CNT})^4 M} t \right) (1 + (-1)^i) \right]}{i\pi \left(EI^{CNT} (i\pi)^4 - \left(\frac{EA}{1-2\nu^*} \alpha^* \theta - T\right) (i\pi L^{CNT})^2 + K (L^{CNT})^4 \right)} \sin \frac{n\pi x}{L^{CNT}} \right\} \quad (11)$$

Eq. (11) is the desired closed-form solution that represents deflection of the SWCNT.

4. Results and discussion

4.1. Convergence criteria

Fig. 2 depicts the convergence criteria based on the number of iteration (i) in the close form solution for

the computation of the deflection of the simply supported SWCNT. The computational time associated with each iteration is shown in Table 1. It is clear that

the solution converges at $i = 3$. Hence, extending the iteration above $i = 3$, will only increase the compu-

Fig. 2. Convergence criteria.

tational time and cost with negligible effect on the improvement of the established solution.

4.2. Effect of the modal number on the deflection of the SWCNT

Fig. 3 illustrates the effects of modal number on the deflection of the simply supported SWCNT. Critical visualization shows that as the modal number increases, the stability of the SWCNT under study decreases. This is because of an increase in the cycles covered by the SWCNT for the same length. These occur because the kernel depends on the modal number.

4.3. Influence of surface uniform pressure on the SWCNT deflection

Figs. 4 and 5 show the influence of external pressure on the deflection of the SWCNT for the first two modes. It is established from the figure that as external pressure increases, there is a corresponding increase in the deflection of the SWCNT. It is shown that the distributed surface pressure when converted into a force will act at the center of the SWCNT span. At that

Table 1

Convergence criteria based on the number of iteration (i) in the close form solution for the SWCNT deflection.

Iteration i	Maximum deflection χ (µm)	Computational time (secs)			
		$n = 1$	$n = 2$	$n = 3$	$n = 4$
1	1.600	0.256784	0.323885	0.435583	0.436961
2	1.601	0.296788	0.323900	0.436781	0.446962
3	3.700	0.456733	0.466385	0.468551	0.486001
10	3.700	1.236734	1.523877	1.835101	1.999921
15	3.700	3.456780	3.723335	3.935513	4.001610

Fig. 3. SWCNT modes with pinned–pinned condition.

Fig. 4. Influence of pressure on deflection for mode 1.

point, the shearing force will be zero while bending moment will be maximum. The maximum moment induces large deflection in the SWCNT that continues to increase as the external pressure increases.

Fig. 5. Influence of pressure on deflection for mode 2.

Fig. 6. Influence of temperature change on deflection for mode 1.

4.4. Effect of change in temperature on the deflection of the SWCNT

Figs. 6 and 7 show the influence of change in temperature on the deflection of the SWCNT for the first two modes. The SWCNT deflection decreases as the change in temperature increases. This is due to thermal expansion coefficient being negative when the SWCNT is considered at room temperature and positive when temperature is very high. Furthermore, at low value of temperature, the flexural rigidity of the SWCNT decreases. This leads to an increase in the flexibility of the SWCNT and consequently increases its deflection.

4.5. Effect of pre-tension on the deflection of the SWCNT

Figs. 8 and 9 depict the influence of pre-tension on deflection of the SWCNT for the first two modes. A careful study of dynamic analysis of the SWCNT

Fig. 7. Influence of temperature change on deflection for mode 2.

Fig. 8. Influence of pre-tension on deflection for mode 1.

shows that as the magnitude of the pre-tension increases, the deflection decreases. This is because the pre-tension as modelled in the governing equation tends to annul the initial static deflection induced in the SWCNT as a result of the reaction at the supports.

4.6. Effect of foundation parameter on the deflection of the SWCNT

Figs. 10 and 11 present the impact of Winkler foundation parameter on the deflection of the single walled carbon nanotube for the first two modes. An increase in the Winkler parameter makes the foundation of the SWCNT to become stiffer and consequently attenuates the deflection of the nanotube.

4.7. Dynamic response of the SWCNT

Figs. 12–15 display the three dimensional dynamic response associated with the SWCNT for the first four

Fig. 9. Influence of pre-tension on deflection for mode 2.

Fig. 10. Effect of foundation parameter on the deflection for mode 1.

Fig. 11. Effect of foundation parameter on the deflection for mode 2.

Fig. 12. Dynamic response of the SWCNT for mode 1.

Fig. 13. Dynamic response of the SWCNT for mode 2.

Fig. 14. Dynamic response of the SWCNT for mode 3.

Fig. 15. Dynamic response of the SWCNT for mode 4.

Table 2

Comparison of present study with Coşkun et al., 2011 [40] Exact method for pinned - pinned condition.

Mode	Mode shape	
	Coşkun et al. [40]	Present study
1	3.14159265	3.14159265
2	6.28318531	6.28318531
3	9.42477796	9.42477796
4	—	12.56637061
5	—	15.70796327

modes. A critical assessment of the figures shows that it is possible to track the behaviour of the CNT at any instance. The dynamic analysis is important as it helps in the quick monitoring and adjustment of the CNT during application.

The present study is reduced to the level of Coşkun et al. [40], and excellent agreements are obtained as shown in Table 2.

4.8. Effect of modal number and length on the frequency of the SWCNT

Figs. 16 and 17 illustrate the influence of modal number and length on the frequency of the SWCNT. A careful study helps in visualizing the effects of these two important parameters on stability of the SWCNT.

Table 3

Comparison of present study with Coşkun et al., 2011 [40] Exact method for pinned - pinned condition for model 1.

Length (nm)	Mode 1	
	Frequency of the Single walled carbon nanotube (THz)	
	Belhadj et al. [26]	Present study
1	0.3	0.3
2	0.2	0.2
3	0.2	0.2
4	0.2	0.2
5	0.2	0.2

Table 4

Comparison of the present study with exact solution of Belhadj et al., 2017 [26] for pinned - pinned condition for mode 2.

Length (nm)	Mode 2	
	Frequency of the Single walled carbon nanotube (THz)	
	Belhadj et al. [26]	Present study
1	1.0	1.0
2	0.6	0.6
3	0.4	0.4
4	0.3	0.3
5	0.2	0.2

Table 5

Comparison of the present study with exact solution of Belhadj et al., 2017 [26] for pinned - pinned condition for Mode 3.

Length (nm)	Mode 3	
	Frequency of the Single walled carbon nanotube (THz)	
	Belhadj et al. [26]	Present study
1	2.5	2.5
2	1.6	1.6
3	0.7	0.7
4	0.5	0.5
5	0.2	0.2

The frequency of SWCNT which is a vital parameter in the study of the SWCNT stability reaches some THz and continues to increase as the modal number increases. This astonishing property enables SWCNT to offer an exceptional optical and mechanical properties although there is always need to dampen the frequency to an application limit. However, an antonymous effect is realized as the length of the SWCNT increases. As a result of their tremendous effects on frequency, these two parameters may be used to annul the effect on each other when one of them is desired based on the requirement of the engineering design and applications.

Tables 3–7 show the comparisons of the present work with the previous past works. The verifications of

Table 6

Comparison of the present study with exact solution of Belhadj et al., 2017 [26] for pinned - pinned condition for mode 4.

Length (nm)	Mode 4	
	Frequency of the Single walled carbon nanotube (THz)	
	Belhadj et al. [26]	Present study
1	—	4.0
2	—	2.5
3	—	0.9
4	—	0.6
5	—	0.2

Table 7

Comparison of the present study with exact solution of Belhadj et al., 2017 [26] for pinned - pinned condition for mode 5.

Length (nm)	Mode 5	
	Frequency of the Single walled carbon nanotube (THz)	
	Belhadj et al. [26]	Present study
1	—	5.0
2	—	4.5
3	—	1.0
4	—	0.8
5	—	0.2

Fig. 16. Effect of modal number and length on frequency.

Fig. 19. Effect of SWCNT mass on frequency.

Fig. 17. Effect of modal number and length on frequency.

Fig. 18. Effect of foundation parameter on frequency.

the present works confirm the correctness and high level of accuracy of the present method in analysis the problem under investigation.

4.9. Influence of foundation variable and mass on SWCNT frequency

Figs. 18 and 19 depict the impact of foundation variable and mass on the excitation frequency of the SWCNT. From the figures, it is obvious that an increase in foundation variable or parameter as well as a reduction in the mass of the single walled carbon nanotube increases the frequency of vibration of the structure.

For both parameters, extreme values should be avoided to prevent instability for very low values of the foundation parameter and over excitation for very high values. Moderate and intelligent choice of the mass of the SWCNT may also be used to annul the above mentioned effects.

4.10. Effect of pre-tension and temperature on the frequency of the SWCNT

Figs. 20 and 21 show the effects of pre-tension and temperature on the dimensional frequency of operation of the present study SWCNT. As the value of pre-tension increases, the operating frequency of vibration of the SWCNT also increases. This is because of the decrease in the initial deflection due to the mass of the SWCNT which consequently increases the operational and dimensional frequency. It is worth noting that the convection of the pre-tension is very important. If the

Fig. 20. Effect of pre-tension on the frequency.

Fig. 21. Influence of temperature change on the frequency.

Fig. 22. Comparison of results.

convention of the pre-tension in the governing equation of the SWCNT in question is reversed, the tension becomes compressive. This increases the initial deflection and consequently increases the frequency and instability of the SWCNT. However, change in temperature has negligible impact on the SWCNT frequency for the range used.

In order to verify the developed closed form solution, the dimensional frequency model obtained in the present study is reduced to the Belhadj et al. [26], SWCNT model. A very good agreement is obtained as shown:

Fig. 22 shows the comparison of results of Belhadj et al. [26], with the results of the present study.

As it is illustrated from the figure, excellent agreements are obtained between the two results.

5. Conclusion

In this paper, analytical investigations of dynamic response of a SWCNT in thermal and pressurized environments have been carried out using Integral transforms. The results of the developed closed-form solution show excellent agreements with the results in literature. Based on the parametric studies, it was realized that as the magnitude of the pressure distribution at the surface of the SWCNT increases, the deflection of the single walled carbon nanotube increases at all the modes while a corresponding increase in the temperature and foundation parameter have an attenuating effect on the deflection of the SWCNT. Furthermore, an increase in the foundation parameter as well as a decrease in the SWCNT mass increases its frequency of vibration. It is envisaged that this work will enhance the use of SWCNT under the influences of thermal and pressure to structural, electrical and mechanical applications.

Nomenclature

$P(x)$	Pressure distribution
A^{CNT}	Area of the SWCNT
I^{CNT}	SWCNT Inertia
EI^{CNT}	Flexural rigidity
t	Time coordinate
T	Axial pre-tension
F_c, F_s	Fourier cosine and sine functions
n	Modal number
P_o	Magnitude of pressure distribution
L^{CNT}	SWCNT length
E	Young modulus

x	Space coordinate
M	Mass of SWCNT
K	Foundation parameter
i	Number of iterations

Greek letters

μ	Pressure coefficient
ν	Poisson ratio
χ	Deflection of the SWCNT
ω	Natural frequency

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