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Analytical approach to investigation of free vibration of thin rectangular plate immersed in fluid, resting on Winkler and Pasternak foundations

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Abstract

Dynamic behaviour of free vibration of rectangular plate is investigated. This study considered an analytical approach to investigation of free vibration of thin rectangular plate immersed in fluid, resting on Winkler and Pasternak foundations.The governing nonlinear partial differential equation is analyzed using two-dimensional differential transform method. The accuracy of the analytical solutions obtained is verified with existing results in literature and confirmed in excellent agreement. Thereafter, the analytical solutions are used for investigation of effect of elastic foundation, fluid and aspect ratio on vibrating plate.From the result, it is observed that, increase elastic foundation parameters increases natural frequency, increase aspect ratio increase natural frequency, influence of fluid decreases the natural frequency of the plate.Hopefully the present study will contribute to existing knowledge in field of vibration.

Keywords

Free vibration; natural frequency; Winkler and Pasternak foundations; virtual added mass; two-dimensional differential transform method;

Cover Page Footnote

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1. Introduction

The use of thin rectangular plate in recent years has gain an increment due to its high strength, ability to withstand high stress, reduction in weight due to its small cross-sectional area and thin thickness while serving the intending purpose. Rectangular plates are used in aircraft fuselage, automobiles, ship deck and hull in naval, railway, buried pipelines, highway and structural constructions in civil engineering. It is therefore important for design engineers to have a broad understanding on the dynamic behaviour of thin plate when submerged in fluid and when it is rested on an elastic foundation. In the study of free vibration of rectangular plates, Zhous and Ji [1] applied exact method to determine the analytical solution. Merneedi and Raonalluri [2] investigated vibration analysis of plate with cut-out using semi-analytical method. Plates resting on foundation is an important subject to the geotechnics engineers. Generally, modelling of foundation with only Winkler type of foundation have proven to be less accurate, due to the deficiencies in shear interaction between the spring elements. This phenomenon led to the adoption of Winkler and Pasternak foundation modelling as a reasonable alternative. Winkler's idealization represents the soil medium as a system of identical but mutually independent, closely spaced, discrete, linearly elastic spring's deformation of foundation, due to the confined applied load on the loaded regions alone. The pressure−deflection relation at any point is obtained with linear relation formula. Meanwhile, for Pasternak foundation, the existence of shear interaction among the spring elements is assumed and this is accomplished by connecting the ends of the springs to the plate, which undergoes only a transverse shear deformation. The load−deflection relationship is obtained by considering the vertical equilibrium of a shear layer. Hence, the pressure−deflection relationship is given by incorporation of shear layer factor to the existing Winkler formula. The Pasternak foundation therefore, accounts for the deficiency part of the Winkler foundation. Wang et al. [3] adopted exact method in the dynamic analysis of rectangular plate with initial stress resting on Pasternak foundation. In a related work of wang et al. [4], an investigation was conducted on rectangular Reddy plates on Winkler-Pasternak foundation. In a further study, Gupta et al. [5] determined the effect of Winkler and Pasternak foundation on vibration analysis of varying thickness orthotropic rectangular plate.

Much attention is being given to plates under fluidic interaction in recent time due to its importance and application in ship building, marine, nuclear and ocean engineering. The study of plate−fluid interaction in engineering is justified for safety and design purpose. Literature survey reveal extensive studies on the characteristic of immersed and submerged plate in fluid. For instance, Lamb [6] carried out an analytical approach into the investigation of fluid-plate coupled system. He determined the natural frequency of clamped circular plate in contact with water using Rayleigh's method, the results obtained were validated with experimental results of Powell et al. [7]. In another work, Dhananjay and Junye [8] investigated the effect of temperature dependent internal source on the onset of convection and heat transfer in a porous layer saturated by a non-newtonian nanofluid. It was observed that the effect of increasing heat source have a destabilizing impact on the stability. In a further work, Yadav et al. [9] studied the effect of hall current on the criterion for the onset of MHD convection in a porous medium layer saturated by a nanofluid. Likewise, Yadav et al. [10] analysed thermal convection in a horizontal layer of a porous medium saturated with a viscoelastic nanofluid. Research into heat convection and fluid is also studied by Refs. [11−13].

Based on previous studies, several methods have been used for solving free vibration of plates, some of which are numerical and analytical. Although, numerical method being very effective in handling nonlinear problems, is still associated with stability and convergence issues, which has corresponding effect on computational time and cost. Similarly, analytical methods suffer a setback in its inability to handle nonlinear problems. However, in the past few years, several semi-analytical methods have been developed to handle nonlinear and linear problems taking into consideration the deficiencies of both numerical and analytical methods, using few iterations in arriving at solutions. Notable examples are Galerkin, Adomian decomposition method (ADM) and Homotopy perturbation method (HPM). The HPM is applied to nonlinear forced vibration of orthotropic circular plate on elastic foundation [14], however, it is associated with difficulty of finding small parameters. Similarly, Ragesh et al. [15] determined the vibration analysis of plates resting on elastic foundation using

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the Galerkin method. Also, Keshmiri et al. [16] applied ADM to free vibration response of nonlinear tappered beam, however, the rigour of finding the Adomian polynomial in ADM is a difficulty that is overcomed by Differential transform method (DTM). The DTM, introduced by Zhou [17] has proven to be very effective in handling vibration problem with little iterations and very accurate results. The edge of DTM over other semi-analytical methods in aspect of precision and accuracy of result, calls for its application in this study.

To author's best knowledge, the analytical approach to investigation of free vibration of thin rectangular plate immersed in fluid, resting on Winkler and Pasternak foundations using two-dimensional differential transformation method has not been attempted in the past works. Therefore, the present study focuses on the use of two-dimensional DTM to investigate free vibration of thin rectangular plate of uniform thickness resting on two-parameter foundations. Analytical solutions obtained were used for parametric studies. Some practical application of the present study in engineering can be seen in water storage tank and culvert cover.

2. Problem formulation and mathematical analysis

The present study considers, a thin rectangular plate of uniform thickness resting on a linear-, nonlinear Winkler- and Pasternak foundations under different edge conditions. The boundary edge of the plate may be free, clamped, simply supported or combination of all, as shown in Fig. 1. The following assumptions according to Refs. [18,19] are made for the development of the governing equation:

1) Normal to the undeformed, mid surface remain straight and normal to the deformed, mid surface is with the same length.
2) Thickness of plate is smaller compared to the other dimensions.
3) Rotary inertia and shear deformation effect is negligible.
4) Normal stresses σ_z in the transverse direction to the plate are considered negligibly small.

Assuming the stresses vary in the $z-direction$over the plate thickness h. Then, the shear force intensities per unit length is defined as [19].

$$Q_x = \int_{-\frac{h}{2}}^{\frac{h}{2}} \sigma_{xz} dz, \; and \; Q_y = \int_{-\frac{h}{2}}^{\frac{h}{2}} \sigma_{yz} dz, \tag{1}$$

Bending moment intensities per unit length is defined as

$$M_x = \int_{-\frac{h}{2}}^{\frac{h}{2}} \sigma_{xz} dz, \; and \; M_y = \int_{-\frac{h}{2}}^{\frac{h}{2}} \sigma_{yz} dz, \tag{2}$$

Also, twisting moment intensities per unit length is defined as

$$M_{xy} = \int_{-\frac{h}{2}}^{\frac{h}{2}} \sigma_{xy} z dz, \; M_{xy} = M_{yx} \; and \; M_{yx} = \int_{-\frac{h}{2}}^{\frac{h}{2}} \sigma_{yx} z dz, \tag{3}$$

Stress—strain Cartesian coordinates is defined as

$$\sigma_x = \frac{E}{1-\nu^2}\left(\epsilon_x + \nu\epsilon_y\right), \; \sigma_y = \frac{E}{1-\nu^2}\left(\epsilon_y + \nu\epsilon_x\right) \; and \; \sigma_{xy} = G\epsilon_{xy}, \tag{4}$$

Substituting Eq. (4) into Eq. (2) gives:

$$M_x = \int_{-\frac{h}{2}}^{\frac{h}{2}} \frac{E}{1-\nu^2}\left(\epsilon_x + \nu\epsilon_y\right) z dz, \; M_y = \int_{-\frac{h}{2}}^{\frac{h}{2}} \frac{E}{1-\nu^2}\left(\epsilon_y + \nu\epsilon_x\right) z dz \; and \; M_{xy} = \int_{-\frac{h}{2}}^{\frac{h}{2}} 2G\epsilon_{xy} z dz, \tag{5}$$

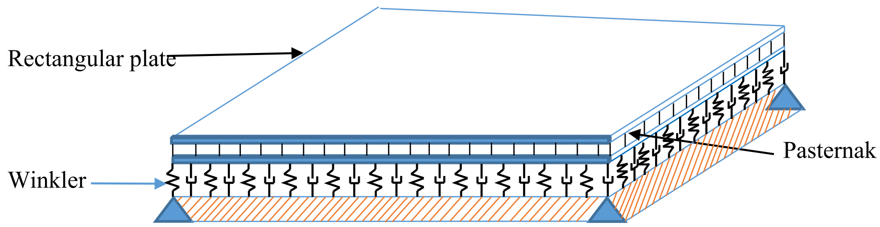

Fig. 1. Showing rectangular plate resting on two-parameter foundations.

Where strains are:

$$\epsilon_x = \frac{\partial \overline{u}}{\partial x}, \epsilon_y = \frac{\partial \overline{v}}{\partial y}, \epsilon_z = \frac{\partial \overline{w}}{\partial z}, \epsilon_{xy} = \frac{1}{2}\left(\frac{\partial \overline{u}}{\partial y} + \frac{\partial \overline{v}}{\partial x}\right), \epsilon_{xz} = \frac{1}{2}\left(\frac{\partial \overline{u}}{\partial z} + \frac{\partial \overline{w}}{\partial x}\right), \epsilon_{yz} = \frac{1}{2}\left(\frac{\partial \overline{v}}{\partial z} + \frac{\partial \overline{w}}{\partial y}\right), \tag{6}$$

The equilibrium in the x-direction without body forces.

$$\frac{\partial \sigma_x}{\partial x} + \frac{\partial \sigma_{xy}}{\partial y} + \frac{\partial \sigma_{xz}}{\partial z} = 0, \tag{7}$$

Multiplying Eq. (7) by z and integrating over the plate thickness gives in x and y direction results to,

$$\frac{\partial Q_x}{\partial x} + \frac{\partial Q_y}{\partial y} + q(x,y) = 0, \tag{8}$$

Which later becomes,

$$\frac{\partial^4 w}{\partial x^4} + 2\frac{\partial^2 w}{\partial x^2 \partial y^2} + \frac{\partial^4 w}{\partial y^4} + \frac{q}{D} = 0, \tag{9}$$

2.1. Formulation of plate coupled fluid governing equation

Similarly, considering a rectangular plate immersed in a fluid as shown in Figs. 2 and 3 respectively the following assumptions are considered for fluid pressure Δp[20],:

1) Vibration is considered linear, plate is of uniform density and thickness

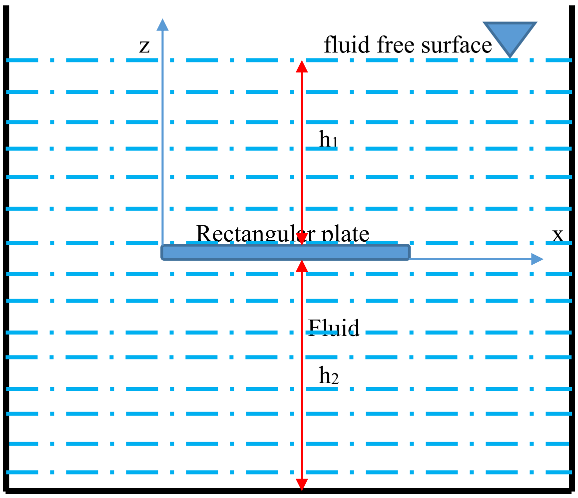

Fig. 2. Showing horizontal submerged plate.

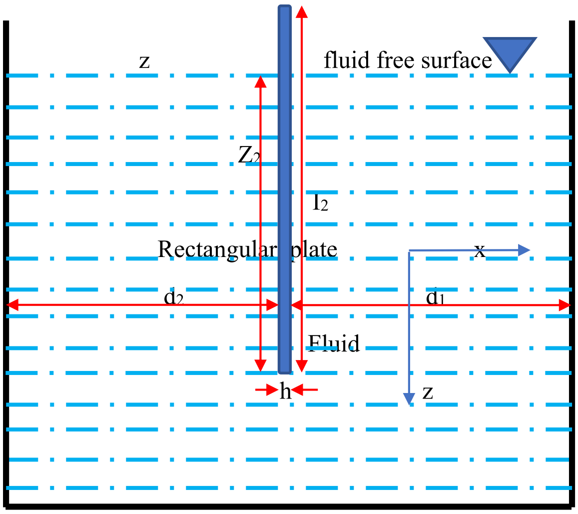

Fig. 3. Showing vertically immersed plate.

2) Flow of fluid is potential, irrotational and homogenous.
3) Fluid is assumed incompressible.
4) Fluid pressure is normal to the plate surface; shear force is ignored, because the flow is inviscid.
5) The motion is considered small.
6) Effect of water dynamic loading has an insignificant effect on mode shapes.
7) The system is conservative.

$$\Delta p = upper\ pressure - \ lower\ pressure$$

$$upper\ pressure = -\frac{\rho_f}{\mu}\left[\frac{1 + Ce^{2\mu h_1}}{1 - Ce^{2\mu h_1}}\right]\frac{\partial^2 w}{\partial t^2} and\ \ lower\ pressure = -\frac{\rho_f}{\mu}\left[\frac{1 + e^{-2\mu h_2}}{1 - e^{-2\mu h_2}}\right]\frac{\partial^2 w}{\partial t^2}, \tag{10}$$

$$\Delta p = m_{add}\frac{\partial^2 w}{\partial t^2}, \tag{11}$$

$where,\ virtual\ added\ mass\ due\ to\ fluid\ for\ horizontal\ submerged\ plate$

$$m_{add} = -\frac{\rho_f}{\mu}\left(\frac{1 + Ce^{2\mu h_1}}{1 - Ce^{2\mu h_1}} - \frac{1 + e^{-2\mu h_2}}{1 - e^{-2\mu h_2}}\right) \tag{12}$$

Similarly, virtual added mass due to fluid for vertically submerged plate

$$m_{add} = -\frac{\rho_f}{\mu}\left(\frac{1+e^{2\mu d_1}}{1-e^{2\mu d_1}} - \frac{1+e^{-2\mu d_2}}{1-e^{-2\mu d_2}}\right) \tag{13}$$

Where $C = \frac{g\mu-\omega^2}{g\mu+\omega^2}, \frac{\partial^2 w}{\partial t^2}$ represents the surrounding fluid inertia, $\omega$is the natural frequency in the vacuum,ρ_f is the fluid density, g represents acceleration due to gravity. According to Kerboua et al. [20], for linear vibration of plate and fluid, C tends to be asymptotical toward -1.μ represents the plane wave number, which represents magnitude of wave motion, it can be determined by

$$\mu = \pi\sqrt{\frac{1}{l_1^2}+\frac{1}{l_2^2}}, \text{ for the horizontally submerged plate} \tag{14}$$

$\mu = \pi\sqrt{\frac{1}{l_1^2}+\frac{1}{z_2^2}}$, for the vertically immersed plate, z_2 is the immersed depth of plate under fluid. Based on the above assumptions, the governing differential equation for thin isotropic rectangular plate as reported by Dumir [21] is:

$$\begin{aligned} &D\nabla^4 w(x,y,t) + k_w w(x,y,t) - k_p w^3(x,y,t) \\ &- g_s\nabla^2 w(x,y,t) \\ &= \rho h \frac{\partial^2 w(x,y,t)}{\partial t^2} + \Delta p, \end{aligned} \tag{15}$$

where in Cartesian coordinates

$$\nabla^4 = \frac{\partial^4}{\partial x^4} + 2\frac{\partial^4}{\partial x^2 \partial y^2} + \frac{\partial^4}{\partial y^4}, \tag{16}$$

Then $w(x,y,t)$is the transverse deflection, $\nabla^4$is the biharmonic operator,k_w, $k_p$and$g_s$are foundation parameters (Winkler's, nonlinear Winkler's and shear parameter of Pasternak foundation respectively), $\rho$and$D$ are mass density and flexural rigidity of the plate respectively, $\nabla^2$is the Laplace operator, $\Delta p$is the fluid dynamic pressure difference on submerged plate in fluid, $\nu$is the Poisson's ratio and $E$is the Young's modulus of the rectangular plate.

$$D = \frac{Eh^3}{12(1-\nu^2)}, \tag{17}$$

In free vibration analyses, based on Kantorovich-type approximations, the resulting solution of Eq. (15) may be represented in the following form [22],

$$w = w(x,y)e^{i\omega t}, \tag{18}$$

where$\omega$is the natural frequency.

Presenting the solution in a more convenient form, these dimensionless parameters are defined:

$$W = \frac{w}{w_{max}}, X = \frac{x}{a}, Y = \frac{y}{b} \tag{19}$$

Therefore, Eq. (15) may be written in terms of the dimensionless parameters as:

$$\frac{\partial^4 W(x,y)}{\partial X^4} + 2\lambda^2 \frac{\partial^4 W(x,y)}{\partial X^2 \partial Y^2} + \lambda^4 \frac{\partial^4 W(x,y)}{\partial Y^4} - \left(\Omega^2 + m_{add} - K_w\right)W(x,y) - K_p W^3(x,y) - g_s\frac{\partial^2 W(x,y)}{\partial X^2} - g_s\frac{\partial^2 W(x,y)}{\partial Y^2} = 0, \tag{20}$$

where, Ω, g_s *and* K_w, K_p are the dimensionless natural frequency, dimensionless Pasternak's Shear stiffness and dimensionless Winkler's normal stiffness and nonlinear Winkler's respectively:

$$\Omega^2 = \frac{\rho h a^4}{D}\omega^2, \quad K_w = \frac{k_w a^4}{D}, \quad g_s = \frac{\overline{g}_s a^2}{D}, \quad M_{add} = \frac{m_{add}a^4}{D}\omega^2, \text{ and } K_p = \frac{a^4 k_p w_{\max}^2}{D} \tag{21}$$

The governing equation of the thin rectangular plate resting on linear, nonlinear Winkler and Pasternak foundations with fluid-interaction in dimensionless form is presented in Eq. (20). Assuming the two opposite edges of the rectangular plate $Y=0$ and $Y=1$ in Fig. 1 to be simply supported, deflection function can be represented as follows to obtain the corresponding nonlinear ordinary differential equation.

$$W = W(X)\sin(m\pi Y), \tag{22}$$

$$\frac{d^4 W(X)}{dX^4} - 2\lambda^2 m^2\pi^2 \frac{d^2 W(X)}{dX^2} - \left(\Omega^2 + M_{add} - K_w - \lambda^4 m^4 \pi^4\right)W(X) - K_p W^3(X) - g_s\frac{\partial^2 W(x,y)}{\partial X^2} = 0, \tag{23}$$

Substitute Eq. (22) into governing differential Eq. (20), we have

2.2. Boundary conditions

The rectangular plate are subjected to the following boundary conditions.

- Simply supported

$$W=\frac{\partial^2 W}{\partial X^2}=0 \;\; at \;\; x=0,1 \quad W=\frac{\partial^2 W}{\partial Y^2}=0 \;\; at \;\; y=0,1, \tag{24}$$

- Free edge supported

$$\frac{\partial^2 W}{\partial X^2}+\nu\frac{\partial^2 W}{\partial Y^2}=\frac{\partial^3 W}{\partial X^3}+(2-\nu)\frac{\partial^3 W}{\partial X\partial Y^2}=0, on \;\; x=0, \tag{25}$$

- Clamped supported

$$W=\frac{\partial W}{\partial X}=0 \;\; on \;\; x=0,1 \quad W=\frac{\partial W}{\partial Y}=0 \;\; on \;\; y=0,1, \tag{26}$$

3. Method of solution: differential transform method

In order to obtain analytical solution to the governing differential equation, two-dimensional differential transformation method is employed. The two-dimensional differential transform of $(m,n)th$ derivative of bivariate function $w(x,y)$in $(x_0,y_0)$is defined as

$$W(m,n)=\frac{1}{m!n!}\left[\frac{\partial^{m+n}w(x,y)}{\partial x^m\partial y^n}\right]_{x=x_0,y=y_0,} \tag{27}$$

Then the inverse is defined as,

$$w(x,y)=\sum_{m=0}^{\infty}\sum_{n=0}^{\infty}W(m,n)(x-x_0)^m(y-y_0)^n, \tag{28}$$

The relation (14) and (15) implies that,

$$w(x,y)=\sum_{m=0}^{\infty}\sum_{n=0}^{\infty}\frac{1}{m!n!}\left[\frac{\partial^{m+n}w(x,y)}{\partial x^m\partial y^n}\right]_{x=x_0,y=y_0,}(x-x_0)^m(y-y_0)^n, \tag{29}$$

Table 1
Operational properties of two-dimensional differential transformation method [23,24].

S/N	Function	Differential transform
1	$w(x,y)\pm f(x,y)$	$W(k,h)\pm F(k,h)$
2	$\alpha w(x,y)$	$\alpha W(k,h)$
3	$\frac{dw(x,y)}{dx}$	$(k+1)W(k+1,h)$
4	$\frac{dw(x,y)}{dy}$	$(h+1)W(k,h+1)$
5	$\frac{d^{m+n}w(x,y)}{dx^m dy^n}$	$\frac{(k+m)!}{k!}\frac{(h+n)!}{h!}W(k+m,h+n)$
6	$w(x,y)f(x,y)$	$\sum_{l=0}^{k}\sum_{p=0}^{h}W(l,h-p)F(k-l,p)$
7	$x^m y^n$	$\delta(k-m,h-n)\Rightarrow \begin{matrix} 1 & k=m,h=n \\ 0 & k\neq m.h\neq n \end{matrix}$
8	$[w(x,y)]^3$	$\sum_{l=0}^{k}\sum_{p=0}^{k-l}\sum_{r=0}^{h}\sum_{s=0}^{h-r}W(l,h-r-s)W(p,r)W(k-l-p,s)$

Application of two-dimensional differential transformation method to the nonlinear equation under investigation.

3.1. Transformation of the nonlinear governing equation

Using the theorem in Table 1, the transformation of the governing Eq. (21) becomes,

$$
\begin{aligned}
&(k+1)(k+2)(k+3)(k+4)W(k+4,h)+2\lambda^2(k+1)(k+2)(h+1)(h+2)W(k+2,h+2)+\\
&\lambda^4(h+1)(h+2)(h+3)(h+4)W(k,h+4)-\left(\Omega^2+m_{add}\right)W(k,h)+K_wW(k,h)+\\
&K_p\sum_{l=0}^{k}\sum_{p=0}^{k-l}\sum_{r=0}^{h}\sum_{s=0}^{h-r}W(l,h-r-s)W(p,r)W(k-l-p,s)-g_s(k+1)(k+2)W(k+2,h)-\\
&g_s(h+1)(h+2)W(k,h+2)=0,
\end{aligned}
\tag{30}
$$

Simplifying Eq. (30) using the theorem in Table 1 results

$$
W(k+4,h)=\frac{-\left(\begin{array}{l}\left(2\lambda^2(k+1)(k+2)(h+1)(h+2)W(k+2,h+2)+\lambda^4(h+1)(h+2)(h+3)(h+4)W(k,h+4)\right)\\ +K_wW(k,h)+K_p\sum_{l=0}^{k}\sum_{p=0}^{k-l}\sum_{r=0}^{h}\sum_{s=0}^{h-r}W(l,h-r-s)W(p,r)W(k-l-p,s)-g_s(k+1)(k+2)\\ W(k+2,h)\end{array}\right)\\ -\left(\Omega^2+m_{add}\right)W(k,h)-g_s(h+1)(h+2)W(k,h+2)}{(k+1)(k+2)(k+3)(k+4)},
\tag{31}
$$

3.2. Transformation of the boundary condition

For a fourth-order differential equation, four conditions are required for the analysis. To avoid repetition of transformations and calculation, simply supported free edge (SSSF) boundary condition is analysed here, while same technique is adopted in analysing the other two boundary conditions, simply supported-simply supported condition (SSSS) and simply supported-clamped edge condition (SSSC). Transforming the conditions in Eq. (24) by using the theorem in Table 1 for condition at $x=0$ we have

Simply supported

$$W(k,h)=0 \qquad W(0,h)=0, \tag{32}$$

$$(k+1)(k+2)W(k+2,h)=0 \qquad W(2,h)=0, \tag{33}$$

While the remaining two conditions required are considered as unknown.

$$W(k,h)=a \qquad W(1,h)=a, \tag{34}$$

$$(k+1)(k+2)W(k+2,h)=b \qquad W(3,h)=\frac{b}{6}, \tag{35}$$

The transformed boundary condition Eq. (24) may be written as

$$
\begin{aligned}
&W(0,h)=0,\\
&W(1,h)=a,\\
&W(2,h)=0,\ \ h=0,\ 1,\ 2,\ 3\ldots N\\
&W(3,h)=\frac{b}{6},
\end{aligned}
\tag{36}
$$

4. The solutions

The transformed non-linear, Eq. (31), is solved iteratively along with the transformed boundary conditions Eq. (36) at $x=0$while the conditions at the edge,$x=1$, are used to determine the unknowns introduced into the boundary condition. Three support conditions are investigated; simply supported at$x=0$, clamped edge support at$x=1$, simply supported at both edges and simply supported at$x=0$, free edge condition$x = 1$. To avoid repetition of transformations and calculation, simply supported edge at $x = 0$and free condition$x = 1$at the edge are presented here:

$$W(5,0) = -\frac{1}{30}\lambda^2 b - \frac{1}{5}\lambda^4 a + \frac{g_s a}{60} - \frac{K_w a}{120} + \frac{(\Omega^2 + m_{add})a}{120} + \frac{g_s b}{120}, \tag{37}$$

$$W(5,1) = -\frac{1}{10}\lambda^2 b - \lambda^4 a - \frac{K_w a}{120} + \frac{(\Omega^2 + m_{add})a}{120} + \frac{g_s b}{120} + \frac{g_s a}{20}, \tag{38}$$

$$W(5,8) = -\frac{3}{2}\lambda^2 b - 99\lambda^4 a - \frac{K_w a}{120} + \frac{(\Omega^2 + m_{add})a}{120} + \frac{g_s b}{120} + \frac{3g_s a}{4}, \tag{39}$$

Applying the definition of DTM, we have

$$W(x,y) = \sum_{j=0}^{m}\sum_{l=0}^{n} w(j,l)x^j y^l \tag{44}$$

The analytical solution of Eq. (15) is written as,

Applying condition at the free edge support$x = 1$,the result is validated by substituting the following parameters, integer $m = 1$, Poisson's ratio$\nu = 0.3$, Winkler foundationk_w and k_p, Pasternak foundation g_s, virtual mass of immersed plate due to surrounding fluid m_{add}, as

$$W(7,0) = -\frac{2}{21}\lambda^2 w_{5,2} - \frac{\lambda^4 b}{210} - \frac{K_w b}{5040} - \frac{K_p a^3}{840} + \frac{(\Omega^2 + m_{add})b}{5040} + \frac{1}{42}g_s\left(-\frac{1}{30}\lambda^2 b - \frac{1}{5}\lambda^4 a - \frac{K_w a}{120} + \frac{(\Omega^2 + m_{add})a}{120} + \frac{g_s b}{120} + \frac{g_s a}{60}\right) + \frac{g_s b}{2520}, \tag{40}$$

$$W(7,4) = -\frac{10\lambda^2 w_{5,6}}{7}\frac{1}{3}\lambda^4 b - \frac{K_w b}{5040} - \frac{K_p a^3}{56} + \frac{(\Omega^2 + m_{add})b}{5040} + \frac{1}{42}g_s\left(-\frac{1}{2}\lambda^2 b - 14\lambda^4 a - \frac{K_w a}{120} + \frac{(\Omega^2 + m_{add})a}{120} + \frac{g_s b}{120} + \frac{g_s a}{4}\right) + \frac{g_s b}{168}, \tag{41}$$

$$W(10,0) = -\frac{2\lambda^2 w_{8,2}}{45} - \frac{\lambda^4 w_{6,4}}{210} + \frac{K_w \lambda^2 w_{4,2}}{37800} - \frac{(\Omega^2 + m_{add})\lambda^2 w_{4,2}}{37800} + \frac{g_s}{90}\left(-\frac{\lambda^4 w_{4,4}}{70} - \frac{g_s \lambda^2 w_{4,2}}{420} - \frac{1}{14}\lambda^2 w_{6,2} + \frac{g_s w_{4,2}}{840}\right) + \frac{g_s w_{6,2}}{2520}, \tag{42}$$

$$W(8,10) = -\frac{2}{3}\lambda^2 w_{8,6} - \frac{1}{3}\lambda^4 w_{6,8} + \frac{K_w \lambda^2 w_{4,6}}{2520} - \frac{(\Omega^2 + m_{add})\lambda^2 w_{4,6}}{2520} + \frac{g_s}{90}\left(-\lambda^4 w_{4,8} - \frac{1}{28}g_s \lambda^2 w_{4,6} - \frac{15\lambda^2 w_{6,6}}{14}\right), \tag{43}$$

zero into Eq. (45). For varying step of y, different

$$W(x,y) = axy^2 + axy + axy^3 + axy^4 + axy^5 + axy^6 + axy^7 + \frac{1}{6}bx^3y + \frac{1}{6}bx^3y^2 + \frac{1}{6}bx^3y^3 + \frac{1}{6}bx^3y^4 + \frac{1}{6}bx^3y^5 + \frac{1}{6}bx^3y^6 + \left(-\frac{1}{30}\lambda^2 b - \frac{1}{5}\lambda^4 a - \frac{K_w a}{120} + \frac{(\Omega^2 + m_{add})a}{120} + \frac{g_s b}{120} + \frac{g_s a}{60}\right)x^5 + \frac{1}{6}bx^3y^7 + \left(\begin{array}{l} -\frac{2}{21}\lambda^2\left(-\frac{1}{5}\lambda^2 b - 3\lambda^4 a - \frac{K_w a}{120} + \frac{(\Omega^2 + m_{add})a}{120} + \frac{g_s b}{120} + \frac{1}{10}g_s a\right) - \frac{\lambda^4 b}{210} - \frac{K_w b}{5040} - \frac{K_p a^3}{840} + \frac{(\Omega^2 + m_{add})b}{5040} + \\ \frac{1}{42}g_s\left(-\frac{1}{30}\lambda^2 b - \frac{1}{5}\lambda^4 a - \frac{K_w a}{120} + \frac{(\Omega^2 + m_{add})a}{120} + \frac{g_s b}{120} + \frac{g_s a}{60}\right) + \frac{g_s b}{2520} \end{array}\right)x^7 + \left(\begin{array}{l} -\frac{2}{7}\lambda^2\left(-\frac{1}{3}\lambda^2 b - 7\lambda^4 a - \frac{K_w a}{120} + \frac{(\Omega^2 + m_{add})a}{120} + \frac{g_s b}{120}\right) - \frac{1}{42}\lambda^4 b - \frac{K_w b}{5040} - \frac{K_p a^3}{280} + \frac{(\Omega^2 + m_{add})b}{5040} + \\ \frac{1}{42}g_s\left(-\frac{1}{10}\lambda^2 b - \lambda^4 a - \frac{K_w a}{120} + \frac{(\Omega^2 + m_{add})a}{120} + \frac{g_s b}{120}\right) \end{array}\right)x^7 y + \left(-\frac{1}{10}\lambda^2 b - \lambda^4 a - \frac{K_w a}{120} + \frac{(\Omega^2 + m_{add})a}{120} + \frac{g_s b}{120} + \frac{g_s a}{20}\right)x^5 y + \left(\begin{array}{l} -\frac{1}{5}\lambda^2 b - 3\lambda^4 a - \frac{K_w a}{120} + \frac{(\Omega^2 + m_{add})a}{120} \\ + \frac{g_s b}{120} + \frac{g_s a}{10} \end{array}\right)x^5 y^2 + \ldots \tag{45}$$

analytical solutions are obtained, substituting $y = 0.05$ the following simultaneous equations are obtained:

$$\left(\frac{78483}{4421} + \frac{8221\Omega^2}{59272}\right)a + \left(\frac{28565}{23881} + \frac{2245\Omega^2}{251942}\right)b = 0, \tag{46}$$

$$\left(\frac{187525}{618} + \frac{24737\Omega^2}{70538}\right)a + \left(\frac{275879}{36839} + \frac{5178\Omega^2}{104887}\right)b = 0, \tag{47}$$

(46) and (47) in matrix form

$$\begin{bmatrix} \left(\frac{78483}{4421} + \frac{8221\Omega^2}{59272}\right) & \left(\frac{28565}{23881} + \frac{2245\Omega^2}{251942}\right) \\ \left(\frac{187525}{618} + \frac{24737\Omega^2}{70538}\right) & \left(\frac{275879}{36839} + \frac{5178\Omega^2}{104887}\right) \end{bmatrix} \begin{Bmatrix} a \\ b \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix}, \tag{48}$$

Non-trivial solution requires that the determinant is equated to zero:

$$\begin{bmatrix} \left(\frac{78483}{4421}+\frac{8221\Omega^2}{59272}\right) & \left(\frac{28565}{23881}+\frac{2245\Omega^2}{251942}\right) \\ \left(\frac{187525}{618}+\frac{24737\Omega^2}{70538}\right) & \left(\frac{275879}{36839}+\frac{5178\Omega^2}{104887}\right) \end{bmatrix} = 0, \tag{49}$$

The Eigenvalue obtained is

$$\frac{5639\Omega^2}{59968}+\frac{125057}{9909} + \frac{\sqrt{1.27\times 10^{15}\Omega^4+8.95\times 10^{17}\Omega^2+9.7\times 10^{20}}}{5.0\times 10^8}, \tag{50}$$

Solving quadratic Eq. (50) gives natural frequencies

$$\Omega = 11.60056007I \tag{51}$$

Substitute the natural frequencies obtained in Eq. (51) into Eq. (50), gives:

$$\begin{bmatrix} -\frac{35487}{38873} & -\frac{457}{151762} \\ \frac{1281994}{5003} & \frac{40568}{47995} \end{bmatrix} \begin{Bmatrix} a \\ b \end{Bmatrix}_{(1)} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix}, \tag{52}$$

Setting $a = 1$, then find$b$,

$$\begin{Bmatrix} a \\ b \end{Bmatrix}_{(1)} = \begin{Bmatrix} 1 \\ -\frac{1011636}{3337} \end{Bmatrix}, \tag{53}$$

The deflection series solution of the governing equation mode 1 is given as,

$$\begin{aligned} w(x,y) = {} & x+\frac{28400x^7}{6833}+\frac{63015x^5}{7174}-\frac{1954927x^7y^7}{2109}-\frac{586868x^7y^5}{1455}-\frac{628043x^7y^6}{990} \\ & -\frac{168606x^3y^7}{3337}-\frac{168606x^3y^6}{3337}-\frac{168606x^3y^4}{3337}-\frac{168606x^3y^5}{3337}-\frac{168606x^3y^3}{3337} \\ & -\frac{168606x^3y^2}{3337}+\frac{1734614x^5y^7}{5847}+xy^5+xy^2+xy^6+xy^3+xy^4-\frac{168606x^3y}{3337} \\ & +xy+xy^7+\frac{307274x^5y^5}{1653}+\frac{15109x^5y^6}{63}+\frac{240018x^5y}{8513}-\frac{55330x^7y}{4923}-\frac{291185x^7y^3}{2459} \\ & -\frac{350191x^7y^4}{1500}+\frac{284183x^5y^3}{3058}+\frac{542281x^5y^4}{3974}+\frac{166309x^5y^2}{2943}-\frac{259991x^7y^2}{5391}-\frac{168606x^3}{3337}+\ldots, \end{aligned} \tag{54}$$

4.1. Nonlinear natural frequency

Applying the Galerkin decomposition method to separate the temporal and spatial part of the displacement function.

$$w(x,t) = \phi(x)u(t), \tag{55}$$

where the generalized coordinate of the system is $u(t)$and the trial function that satisfies the natural boundary condition and geometric is $\phi(x)$.

Applying one-parameter Galerkin solution on Eq. (56) to Eq. (23). We have,

$$\int_0^1 R(x,t)\phi(x)dx, \tag{56}$$

Where

$$\begin{aligned} R(x,t) = {} & \frac{\partial^4 W(X,t)}{\partial X^4} - 2\lambda^2 m^2 \pi^2 \frac{\partial^2 W(X,t)}{\partial X^2} \\ & -\left(M_{add}-K_w-\lambda^4 m^4 \pi^4\right)W(X,t) \\ & -g_s\frac{\partial^2 W(x,y)}{\partial X^2} - K_p W^3(X,t) = \frac{a^4\rho h}{D}\frac{\partial^2 w(X,t)}{\partial t^2}, \end{aligned} \tag{57}$$

We have

$$M\ddot{u}_s(t)+Ku_s(t)+Vu_s^3(t)=0, \tag{58}$$

Where

$$M = \int_0^1 \phi\left(-\frac{a^4\rho h}{D}\phi\right)dx, \tag{59}$$

$$K=\int_0^1\phi\left(\frac{d^4\phi}{dx^4}-2\lambda^2m^2\pi^2\frac{d^2\phi}{dx^2}-g_s\frac{d^2\phi}{dx^2}-\left(M_{add}-K_w-\lambda^4m^4\pi^4\right)\phi\right)dx, \tag{60}$$

$$V=\int_0^1\phi\left(-k_p\phi^3\right)dx, \tag{61}$$

4.2. The initial and boundary conditions

The rectangular plate may be subjected to any of the following boundary conditions.

- Clamped–Clamped support

$$\phi(x)=cosh\beta_nx-cos\beta_nx-\left(\frac{sinh\beta_nL+sin\beta_nL}{cosh\beta_nL-cos\beta_nL}\right)(sinh\beta_nx-sin\beta_nx) \tag{62}$$

where$\beta_n$are the roots of the equation

$cos\beta_nLcosh\beta_nL=1$

The initial and boundary conditions are:

$$w(0,x)=a,\ \dot{w}(0,x)=0, \tag{63}$$

$$w(0,t)=w'(0,t)=0,\ \ w(L,t)=w'(L,t)=0$$

Alternatively, polynomial function of the form Eq. (64) can be applied for this type of support system.

$$\phi(x)=25.20\times\left(x^2-2x^3+x^4\right); \tag{64}$$

- Simply- Supported:

$$\phi(x)=sin\beta_nx \tag{65}$$

$$sin\beta L=0\Rightarrow\beta_n=\frac{n\pi}{L}$$

The initial and boundary conditions are

$$w(0,x)=a,\ \dot{w}(0,x)=0, \tag{66}$$

$$w(0,t)=w''(0,t)=0,\ \ w(L,t)=w''(L,t)=0$$

Alternatively, polynomial function of the form Eq. (67) can be applied for this type of support system.

$$\phi(x)=3.20\times\left(x-2x^3+x^4\right); \tag{67}$$

4.3. Determination of natural frequencies

The dynamic response of the structural analysis is carried out under the transformation:

$$\tau=e^{iwt}, \tag{68}$$

Applying Eq. (68) on Eq. (58), we have

$$M\omega^2\ddot{u}(\tau)+Ku(t)+Vu_s^3(t)=0, \tag{69}$$

In order to find the periodic solution of Eq. (69), assume an initial approximation for zero-order deformation as;

$$u_0(\tau)=A\cos\tau, \tag{70}$$

Substitute Eq. (70) into Eq. (69), we have

$$-M\omega_0^2A\cos\tau+KA\cos\tau+VA^3\cos^3\tau=0, \tag{71}$$

Which gives

$$-M\omega_0^2A\cos\tau+KA\cos\tau+VA^3\left(\frac{3\cos\tau+\cos3\tau}{4}\right)=0, \tag{72}$$

Collecting like terms, we have:

$$\left(KA-M\omega_0^2A+\frac{3VA^3}{4}\right)\cos\tau-\frac{1}{4}VA^3\cos3\tau=0; \tag{73}$$

Eliminating the secular term, we have:

$$\left(KA-M\omega_0^2A+\frac{3VA^3}{4}\right)=0, \tag{74}$$

Thus, zero-order nonlinear natural frequency becomes:

$$\omega_0\approx\sqrt{\frac{K}{M}+\frac{3VA^2}{4M}}, \tag{75}$$

Therefore, the ratio of zero-order nonlinear natural frequency, $\omega_0$to the linear frequencyω_b;

$$\frac{\omega_0}{\omega_b}=\sqrt{1+\frac{3VA^2}{4K}}, \tag{76}$$

5. Results and discussion

The analytical solutions of thin rectangular plate immersed in fluid and resting on Winkler and Pasternak foundation are analysed using two-dimensional differential transformation method. Square aluminium isotropic rectangular plates $l_1 = 1$ and $l_2 = 1$ are considered. It is observed that, the more the iteration increases, the more the computation time increases and the higher mode natural frequency attained. This is peculiar to vibration problem. The material constants for the simulation and parametric studies are taken from Ref. [25] and presented in Table 2. The first-three mode natural frequencies comparison of present results, with that of previous works [26,28] are illustrated in Tables 3—5. Based on the results presented in Table 3, it can be concluded that the results obtained are in good harmony with the published work. Furthermore, variation of elastic foundation and aspect ratio on natural frequencies are shown in Table 6. Figs. 4—6 presented the variation effects of foundation parameters on natural frequency. Also, the variation of natural frequencies with aspect ratio of the rectangular plate are shown in Figs. 7—9. The first-three modal shape of the thin rectangular plate are illustrated in Figs. 10—12. Based on the results, it is deduced that the natural frequency vary linearly and increase with increase in foundation parameters and aspect ratio. It is also observed from the results that, submerging the plate in water decreases the natural frequency.

Table 2
Showing parameters.

Material density	Young's modulus	Density of fluid	Poison's ratio	gravity	Reservoir tank dimension	Plate thickness
ρ(kg/m^3)	E (Gpa)	ρ_f (kgm^{-3})	ν	g	(m)	h (m)
2700	69	1000	0.3	9.8	5 × 5 x 5	0.01

Table 3
Validation of results with exact method.

Edge Condition/Dimensionless natural frequency	Simply Supported (SSSS)		Simply Supported-clamped (SSSC)		Simply Supported-free (SSSF)	
	Aspect ratio λ = 1					
	Leissa [26]	Present	Leissa [26]	Present	Leissa [26]	Present
Ω_1	19.7392	19.7392	23.6463	23.6463	11.6845	11.600
Ω_2	49.348	49.348	58.6464	58.6464	27.7563	27.7563
Ω_3	98.696	98.6822	113.2281	113.179	61.8606	61.8606

Table 4
Validation of results for first-three modes.

Edge Condition/Dimensionless natural frequency	Aspect ratio λ	Simply Supported (SSSS)		% Diff.	Simply Supported-clamped (SSSC)		% Diff.
		Leissa [26]	Present		Leissa [26]	Present	
Ω_1		19.7392	19.7392	0	23.6463	23.6463	0
Ω_2	1	49.348	49.348	0	58.6464	58.6464	0
Ω_3		98.696	98.682	0.014	113.228	113.179	0.049

Table 5
Validation of results for first-three modes.

Edge Condition/Dimensionless natural frequency	Aspect ratio λ	Simply Supported-free (SSSF)		% Difference
		Leissa [26]	Present	
Ω_1	1	11.6845	11.6	0.0845
Ω_2		27.7563	27.7563	0
Ω_3		61.8606	61.8606	0

Table 6
Showing Variation of Aspect ratio and foundation coefficient.

Edge Condition	Natural frequency	$\lambda = 0.5$			$\lambda = 1.5$		
	Mode	$k_w = 10$	$k_w = 50$	$k_w = 100$	$k_w = 10$	$k_w = 50$	$k_w = 100$
SSSS	Ω_1	12.7358	14.2198	15.8809	32.2317	32.8464	33.5989
	Ω_2	42.0649	42.5376	43.1214	61.766	62.089	62.4903
	Ω_3	91.3334	91.5521	91.8248	111.0663	111.2461	111.4707
SSSC	Ω_1	17.6179	18.7187	20.0097	35.1935	35.7573	36.4497
	Ω_2	52.1938	52.5756	53.049	69.9843	70.2695	70.6244
	Ω_3	106.4681	106.6556	106.8899	124.635	124.7947	124.9952
SSSF	Ω_1	5.1255	8.1407	10.7829	24.2175	25.0297	26.0093
	Ω_2	19.0847	20.1053	21.3125	41.2952	41.7767	42.3709
	Ω_3	53.1197	53.4948	53.9602	75.885	76.1481	76.4756

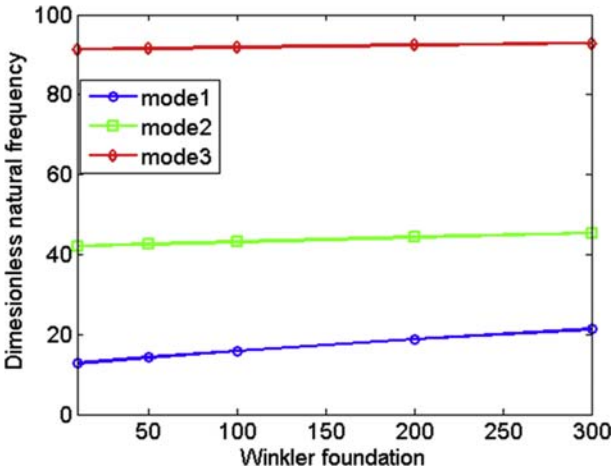

Fig. 4. Variation of foundation parameter on SSSS Boundary conditions.

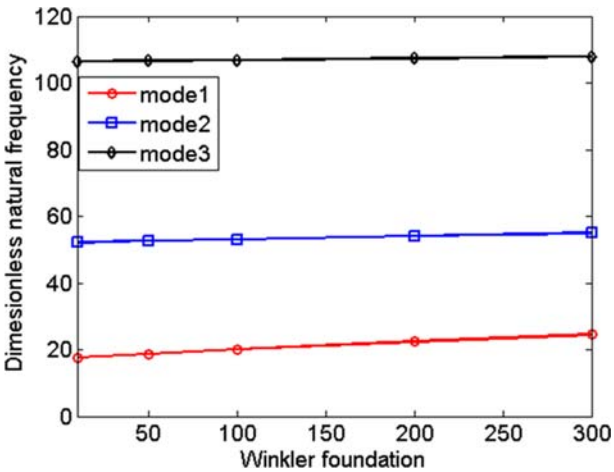

Fig. 5. Variation of foundation parameter on SSSC Boundary conditions.

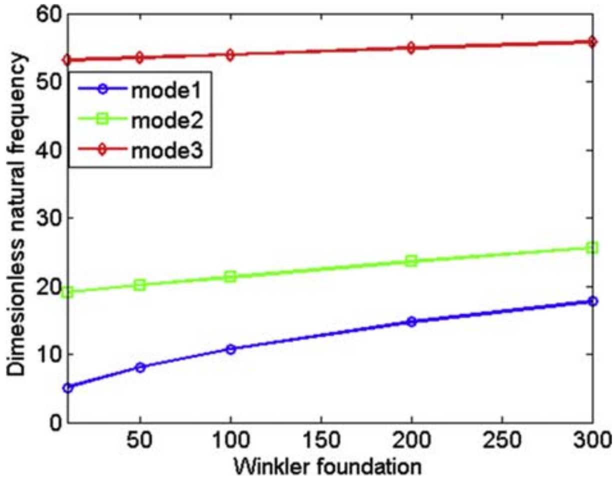

Fig. 6. Variation of foundation parameter on SSSF Boundary conditions.

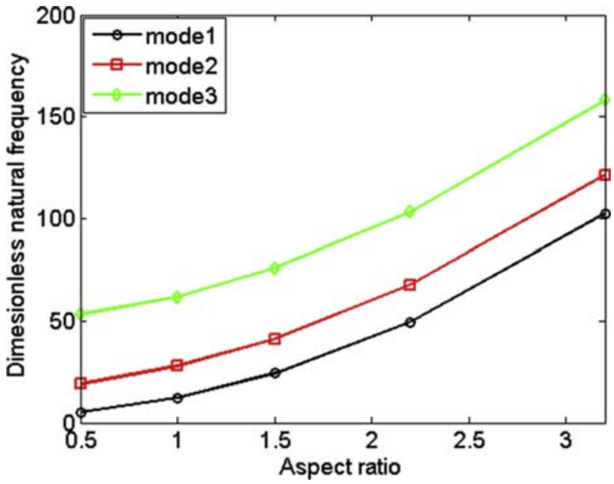

Fig. 7. Variation of Aspect ratio on SSSF boundary condition.

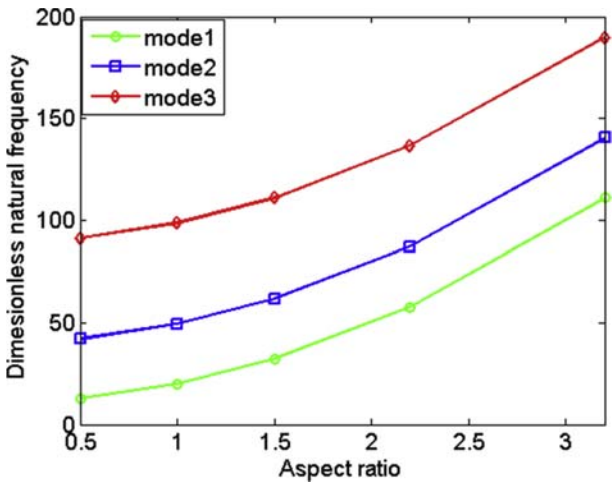

Fig. 8. Variation of Aspect ratio on SSSS boundary condition.

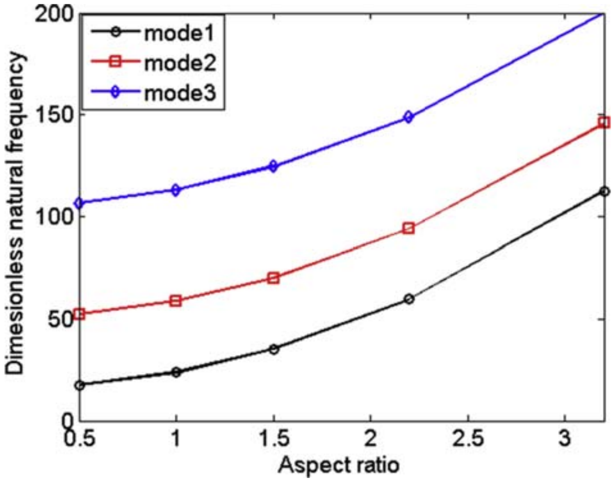

Fig. 9. Variation of Aspect ratio on SSSC boundary condition.

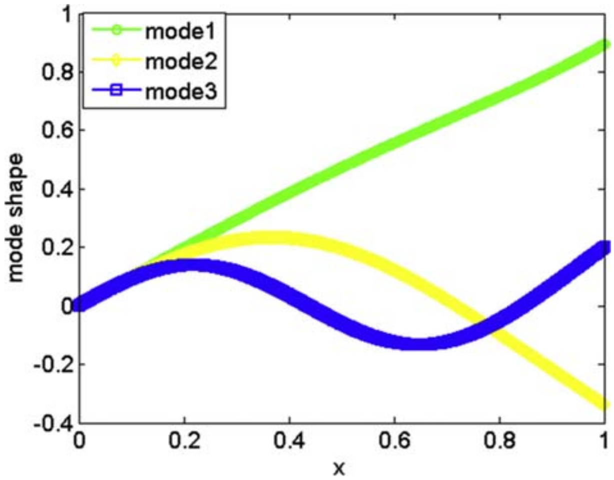

Fig. 10. First-three mode shapes under SSSF boundary supports.

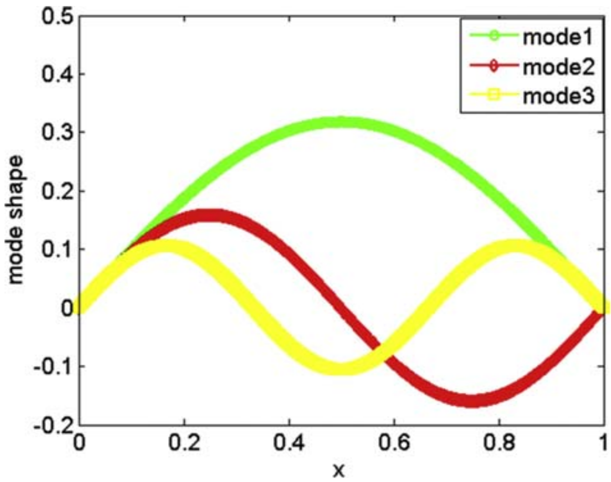

Fig. 11. First three mode shapes under SSSS boundary supports.

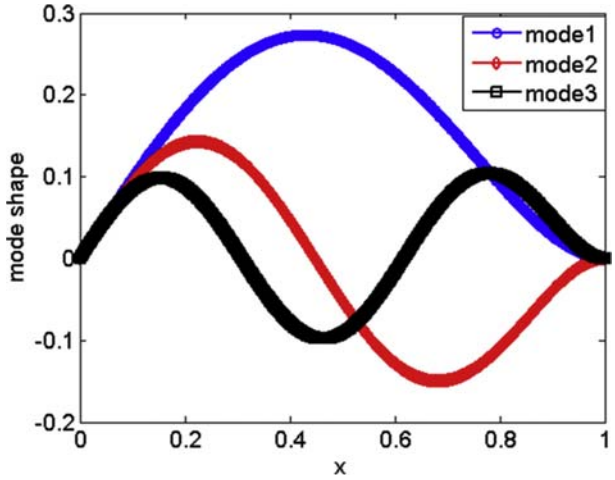

Fig. 12. First three mode shapes under SSSC boundary supports.

5.1. *Effect of foundation parameter on natural frequency*

To investigate the effect of foundation parameters on natural frequency, the derived governing equation is analysed and the natural frequencies obtained are presented in Table 6. In addition, to have the understanding of the numerical results, the natural frequencies are presented in Figs. 4–6. It clearly shows that, for the three boundary conditions SSSS, SSSC, SSSF the foundation parameter has considerable influence on natural frequency, i.e., increasing the values of the foundation parameter increases the natural frequency. This is as a result of increase in stiffness, which is directly proportional to the natural frequency. Therefore, the observation is justified based on the theorem of classical vibration.

5.2. *Effect of variation of aspect ratio on natural frequency*

In order to illustrate the effect of aspect ratio on the natural frequency, the mathematical model presented in Eq. (15) is analysed and the natural frequencies obtained are shown on Table 6. The influence of aspect ratio on the natural frequency is examined and scrutinized in Figs. 7–9, which illustrates the variation of the natural frequencyΩ, with the aspect ratioλ. Based on the presented results, it can be deduced from the figures that, aspect ratio has a direct influence on natural frequency. It is also shown that, increase value of aspect ratio leads to increase in the natural frequency. This can be attributed to the fact that an increase in aspect ratio amount to an increase in stiffness of the plate.

5.3. *Effect of submerging the plate in fluid*

Table 7 illustrates the effect of fluid in contact with vibrating isotropic rectangular plate. The fluid in consideration is water. From the results presented, it can be deduced that the natural frequency of the plate when submerged in fluid decreases as compared to condition without fluid. This is as a result of increase in the kinetic energy of the entire system, i.e., when the kinetic energy of the fluid is added to that of the plate without increase in strain energy. This becomes an added mass to the vibrating plate. Consequently, as the plate vibrates, its mass increases by the addition of fluid mass, hence, the natural frequency decreases. From the theorem of classical vibration, mass has an indirectly proportional effect to natural frequency.

Table 7
Showing comparison of plate natural frequency when immersed in fluid to when outside the fluid.

Edge condition/dimensionless natural frequency	Simply Supported (SSSS)		Simply Support-Clamped (SSSC)		Simply Support-Free (SSSF)	
	Outside fluid [27]	In fluid	Outside fluid [27]	In fluid	Outside fluid [27]	In fluid
Ω_1	19.7392	18.0635	23.6463	22.2667	11.7195	8.5547
Ω_2	49.3481	48.7020	58.6465	58.1038	27.7563	26.5908
Ω_3	99.3042	98.3607	113.522	112.8986	61.8606	61.3465

Again, it is also observed from Figs.10−12 that, the mode shapes for the first-three natural frequencies considered, has no significant changes when compared to the condition when the plate is not submerged in fluid. This finding is authenticated with established reports presented in literature [6].

5.4. Effect of Pasternak foundation on nonlinear natural frequency

To obtain the nonlinear natural frequency, the governing equation is transformed into Duffing equation and the frequency ratio determined. The variation of nonlinear frequency with amplitude is shown for the fundamental mode of vibration in Figs. 13−15. The frequencies are calculated, taking into consideration the value of aspect ratioλ as unity. It is deduced from Fig. 13 that, as nonlinear Winkler foundation increases, the nonlinear vibration frequency ratio decreases. This is a case of softening nonlinearity properties. Fig. 14 depicts the variation of the frequency ratio of the plate under different boundary conditions. From the result, it is observed that frequency ratio is higher in clamped−clamped supported than in simple−simple supported due to the higher stiffness of clamped boundary condition. However, it is noticed from the figures that, the nonlinear frequency is a function of amplitude, since the larger the amplitude, the more significant the discrepancies between the linear and nonlinear frequency.

Fig. 15 illustrate the comparisons and the combine effect of Winkler and Pasternak foundations. Likewise,

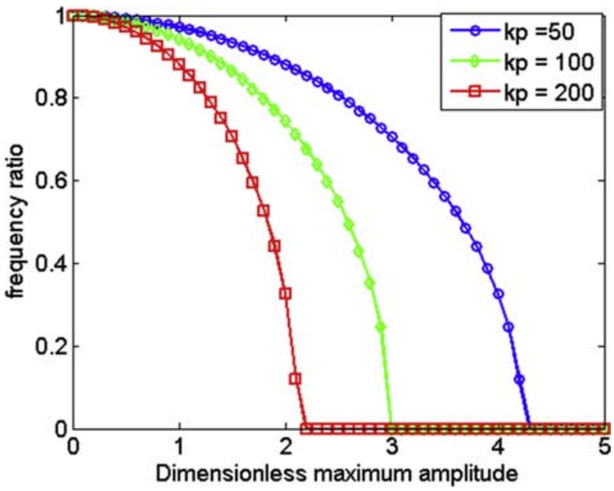

Fig. 13. Effect of Pasternak foundation on the nonlinear amplitude frequency response curve of the isotropic rectangular plate.

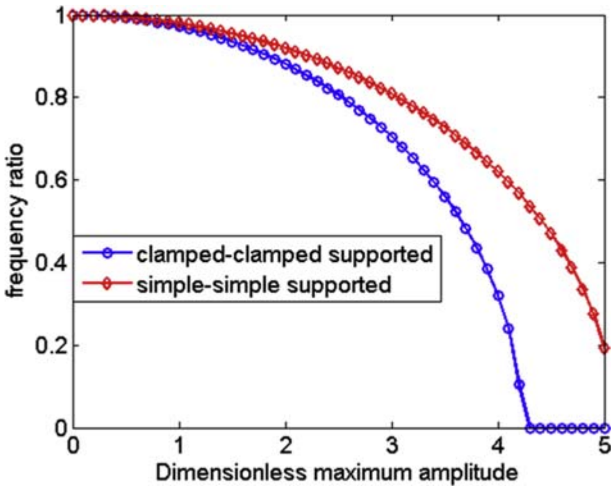

Fig. 14. Effect of boundary conditions on the nonlinear amplitude frequency response curve of the isotropic rectangular plate.

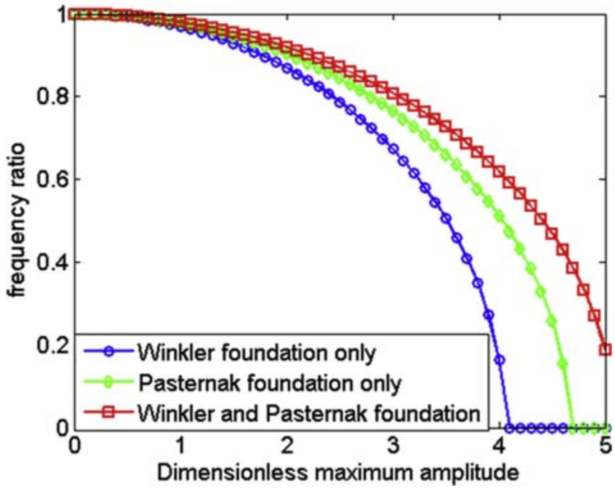

Fig. 15. Effect of Winkler and Pasternak foundation parameters on the nonlinear natural frequency.

it is establishes the fact that nonlinear natural frequency has higher values when the plate is resting on Pasternak foundation. Therefore, parameters can be used to control the nonlinearity of the plate.

6. Conclusion

The present study provides analytical approach to investigation of dynamic behaviour of isotropic rectangular plates resting on Winkler and Pasternak foundations when submerged in a fluid. The governing nonlinear partial differential equation is solved without conversion to nonlinear ordinary differential equation (ODE), in order to eliminate any form of error that might be introduced. The nonlinear partial differential equation was analysed using two-dimensional differential transformation method. The nonlinear fundamental natural frequencies are also determined. The accuracy of the analytical solutions obtained were ascertained by comparing the obtained results with the results from previous studies. The obtained analytical solutions were used to examine the effects of foundation parameters, fluid and aspect ratio. From the parametric studies, the following conclusions were drawn:

1) As the value of elastic foundation parameter increases, the foundation of the plate become stiffer and consequently increases the natural frequency.
2) Increase in aspect ratio results in an increase in stiffness of the plate, which invariably leads to increase in natural frequency.
3) Submerging the plate in fluid lowers the natural frequency. This is due to the added mass effect.
4) Mode shape remain the same as in fluid and without fluid.
5) From the result, it is observed that frequency ratio is higher in clamped—clamped than simple—simple supported condition due to the higher stiffness of clamped boundary conditions. Also, it is noticed that, the nonlinear frequency is a function of amplitude, since the larger the amplitude the more significant the discrepancies between the linear and nonlinear frequency.
6) It is observed that, as nonlinear Winkler foundation increases, the nonlinear vibration frequency ratio decreases. This is a case of softening nonlinearity.

From the present study it can be concluded that DTM is a very powerful tool and at the same time very robust when dealing with eigenvalue problems involving rectangular plates.

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Nomenclature

t	Time
x, y	space coordinate along the dimension of thin plate

EI Young modulus of elasticity
m Integer
$\frac{d}{dx}, \frac{d}{dy}$ Differential operator
k_w Winkler foundation parameter
k_p Nonlinear Winkler foundation parameter
g_s Pasternak foundation parameter
m_{add} Virtual added mass
Δp Fluid dynamic pressure difference

Symbols
ν Poisson' ratio
Ω Frequency of vibration
$w(x,y)$ Deflection of rectangular plate
ρ Mass density
λ Aspect ratio

Data availability

The data used to support the findings of this study are available with the corresponding author upon request.

Conflicts of interest

The author declares that there are no conflict of interest regarding the publication of this paper.

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