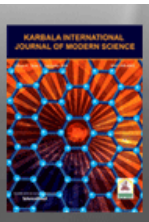

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Study of Deformation Parameters (β2, δ) For 18,20,22,24,26,28Ne isotopes in sdpf shell

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Study of Deformation Parameters (β2, δ) For 18,20,22,24,26,28Ne isotopes in sdpf shell

Abstract

The quadrupole deformation is basic to study the shape transitions; it is possible to predict many important properties of even-even nuclei as a function of the deformation parameter. The deformations of nuclei are important for understanding their shapes prolate or oblate. The quadrupole deformation parameters were calculated by the transition probability B (E2) for $^{18, 20, 22, 24, 26,28}$Ne isotopes, were adopted different interactions three. The calculations are performed with the Bohr-Mottelson (B-M) effective charges, which represent the effect of the core-polarizations. Also, the deformation parameters were calculated for different nuclei and adopted two methods of calculation: from reduced the transition probability (B2) and from the intrinsic quadrupole moment. In calculations, the Shell model is adopted.

Keywords

Quadrupole deformation parameter; Effective charges; Core; Intrinsic quadrupole moment; Shell model.

1. Introduction

The calculation of deformed nuclei was an old problem in the structure of nuclear physics. The mean-field descriptions in the intrinsic frame have been favored; as they take interest natural of the spontaneous collapse of rotational symmetry [1].

The spherical nuclear model becomes unstable when nucleons (protons and neutrons) or gaps (holes) in non-filled shells are increased. The outer nucleons (protons and neutrons) interact with the residual interaction. The interaction results in a correlated motion of the particles leading to a non-spherical nuclear shape. Therefore, the nuclei with many protons and neutrons in unfilled shells have non spherical and ellipsoidal shapes [2]. The Shell model hypothesizes stated that there is a nucleus (core) consisting of pairs of protons and neutrons. This nucleus may be spherically symmetric where it gives rise to the spherically symmetric independent particle model, as seen in the Nilsson model [3]. It is also indicated as a deformed independent particle model. This model was restricted to nuclei with axially symmetric quadrupole deformations. The quadrupole deformations use the deformation parameter (β_2). The deformation parameter is responsible for the prolate deformation at positive values and the oblate deformation at negative values. The nuclei are also classified into oblate and prolate when the quadrupole deformations are ($Q^{'} < 0$) and ($Q^{'} > 0$), respectively.

Alex Brown [4] stated a set of code of NuShellX@MSU using Hamiltonians data files and the model spaces to generate input for NuShellX. The code also converts the NuShellX output into tables and figures for energy levels, beta and gamma decays [4]. The calculation of transition rate B(E2↑) is an index of collectivity and is much more useful than the level energies, where the experimental methods of deduced B(E2↑) values from the excited-state energies [5] are based on data obtained from nuclei close to the valley of stability.

The quadrupole deformation parameter is the basic to the measurement of the shape transitions. The deformation parameters of nuclei are important to understand their shapes (oblate and prolate).

2. Theory

The operator electric transition *O (E2)* is sufficient to know the charge distribution, which can be defined as [6]:

$$O(EJ) = \sum_{k=1}^{A} e(k) r^{k}(k) Y_j(\overrightarrow{r}(k)) \tag{1}$$

where, e(k) represents the charge of nucleon numbered k, i.e. e(k) = 0 for the neutron and e(k) = e for the proton. The reduced matrix element $\langle J_f | \widehat{O}_J || J_i \rangle$ can be calculated by Ref. [7].

$$\langle J_f || \widehat{O}_J || J_i \rangle = \sum_{jj'} OBDM\left(J_i, J_f, J, j, j'\right) \langle j' || \widehat{O}_J || j \rangle$$

where $j^{'}, j$ represent the final and the initial single-particle states, (OBDM) denotes one-body density matrix and $\langle j^{'} || \widehat{O}_J || j \rangle$ represents single-particle matrix elements.

The equation of the model space matrix elements is formulated as follows [7],

$$\langle J_f || \widehat{O}_J || J_i \rangle = \sum_{t_z} e^{eff}(t_z) \langle J_f || \widehat{O}_J || J_{iMS} \rangle$$

where e^{eff} represents the effective charges to the neutrons and the protons only and it is active in restriction of the model space. In the structure of the nuclear physics, the quadrupole moment of a state of angular momentum J is defined as the expectation value in the state of $M = J$. It refers to the spectroscopic or static quadrupole moment [6].

$$Q(J{=}2) = \sqrt{\frac{16\pi}{5}} \sqrt{\frac{J(2J-1)}{(J+1)(2J+1)(2J+3)}} \langle J || O(E2) || J \rangle \tag{2}$$

The calculation of a quadrupole moment of the initial and the final state wave function must be taken identical. The spectroscopic quadrupole moment Q is defined as the moment in the state of $M = J$. It is the maximum value of the moment that one can observe in the laboratory system as shown in the following equation [6]:

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$$Q=\frac{2J-1}{2(J+1)}Q' \tag{3}$$

where Q' represents the intrinsic quadrupole moment, i.e. it is defined with respect to the axis of the symmetry of the charge distribution. The semi-classical derivation shows that the quadrupole moment is partially ($Q < Q'$) due to the precession of the nuclear system in the laboratory system.

The B(E2) transition rate (reduced electric transition probability) can be defined as shown by Ref. [8]:

$$B(EJ)=\langle\frac{J_f||\widehat{O}_J||J_i\rangle}{2J_i+1}^2 \tag{4}$$

where the transition is from the initial state J_i to the finial state J_f.

Also, $Q^{'}$ can be calculated by Refs. [9,10]:

$$Q'=\sqrt{\frac{16\pi}{5}}[B(E2)]^{1/2} \tag{5}$$

and [10]:

$$Q'=\frac{(I+1)(2I+3)}{(3K^2-1(I+1))}Q \tag{6}$$

where Q represents the electric quadrupole moment in the laboratory system, I represents the spin of the state of the member of rotational band based on the nucleus ground state [5] and K represents a projection of I's on the symmetry axis [5].

Calculations of the quadrupole deformation parameters adopted two methods: from B(E2) as shown by Ref. [9]:

$$\beta_2=\left(\frac{4\pi}{3ZR^2}\right)\left[B(E_2)\uparrow\frac{e^2fm^4}{e^2}\right]^{1/2} \tag{7}$$

where $R_0=1.2A^{1/3}\,fm$, Z represents the atomic number, and from the $Q^{'}which$is given by Refs. [5,10]:

$$\delta=\frac{0.75Q'}{\langle Zr^2\rangle} \tag{8}$$

The r^2 value is evaluated using the following equations [5,7]:

$$\langle r^2\rangle=0.6\left(1.2A^{1/3}\right)^2 \quad (A>100) \tag{9}$$

$$\langle r^2\rangle=0.6R_\circ^2\left(1+10\,/\,3(\pi a_\circ/R_\circ)^2\right)\left(1+(\pi a_\circ/R_\circ)^2\right) \quad (A\le 100) \tag{10}$$

where $a_\circ = 0.55$ fm and $R_\circ = 1.07A^{1/3}fm$. The deformation parameter δ is related to the experimental quadrupole moment [11].

$$\delta\approx 0.95\beta_2 \tag{11}$$

The measured deformations of heavy nuclei have $Q > 0$ (prolate shape) and are strongly deformed. While the deformation parameters for the lighter nuclei are somewhat large [11].

3. Results and discussion

Comparing the theoretical quadrupole moments, which are significant values for deformed nuclei, with the experimental data give a chance to test the nuclear models [12]. The B(E2) for isotopes ^{18,20,22,24,26,28}Ne were calculated using different effective interactions and the Bohr-Mottelson (B-M) effective charges [11]. The calculations were performed using the NushellX @ MSU code [4] to calculate the one-body density matrix (OBDM). The B(E2) values were calculated using the *usdb* interaction (Universal *sd*-shell interaction B) [13] for isotopes ^{18,20,22,24,26,28}Ne and tabulated in Table (1). The quadrupole deformation parameters (β_2) were calculated from B(E2) values according to equation (7) and the results were compared with the experimental data [14]. The nucleus of ^{18}Ne has magic

Table 1
The quadrupole deformation parameter for isotopes ^{18,20,22,24,26,28}Ne with *usdb* interaction [13]. The calculation of quadrupole deformation parameter β_{2th} for each isotope was obtained from the adopted values of B(E2) [14].

A,N $_{10}$Ne	$(E_x)_{\text{the}}$ (MeV)	$(E_x)_{\text{ex}}$ (MeV)	$B(E2)_{\text{B-M}}$ (e^2fm^4)	$\boldsymbol{B(E2)}_{ex.}$[13](e^2fm^4)	β_{2th} *usdb int.*	β_{2ex} [13]
18, 8	1.99	1.88	123 (49)	243 (16)	0.4699 (28)	0.661 (21)
20, 10	1.74	1.63	300.6 (10)	333 (16)	0.6848 (5)	0.72 (18)
22, 12	1.36	1.27	279.1 (22)	229.8 (42)	0.6191 (9)	0.561 (51)
24, 14	2.11	1.98	191.4 (33)	$143\left(\frac{+57}{-24}\right)$	0.4839 (17)	$0.418\left(\frac{+0.84}{-0.35}\right)$
26, 16	2.06	2.01	147 (6)	155 (32)	0.4020 (2)	0.413 (43)
28, 18	1.62	1.30	180.2 (32)	136 (32)	0.4236 (16)	0.367 (0.31)

Table 2
The quadrupole deformation parameter for isotopes ^{18,20,22,24,26,28}Ne with *cw* interaction [15].The calculated quadrupole deformation parameter β_{2th} for each isotope was obtained from the adopted values of B(E2) [14].

A,N $_{10}$Ne	$(E_x)_{\text{the}}$ (MeV)	$(E_x)_{\text{ex}}$ (MeV)	$B(E2)_{\text{B-M}}$ (e^2fm^4)	$B(E2)_{ex.}$[13](e^2fm^4)	β_{2th} *cw int.*	β_{2ex} [13]
18, 8	2.00	1.88	119.49 (51)	243 (16)	0.4631 (30)	0.661 (21)
20, 10	1.74	1.63	300.4 (10)	333 (16)	0.6845 (5)	0.72 (18)
22, 12	1.39	1.27	278.8 (21)	229.8 (42)	0.6189 (8)	0.561 (51)
24, 14	2.27	1.98	193.4 (34)	$143\left(\frac{+57}{-24}\right)$	0.4864 (17)	$0.418\left(\frac{+0.84}{-0.35}\right)$
26, 16	2.28	2.01	281 (81)	155 (32)	0.5558 (34)	0.413 (43)
28, 18	1.66	1.30	142 (4)	136 (32)	0.3761 (2)	0.367 (31)

Table 3
The quadrupole deformation parameter for the isotopes ^{18,20,22,24,26,28}Ne with *su3* interaction [16].The calculated of quadrupole deformation parameter β_{2th} for each isotope was obtained from the adopted values of B(E2) [14].

A $_{10}$Ne	$(E_x)_{\text{the}}$ (MeV)	$(E_x)_{\text{ex}}$ (MeV)	$B(E2)_{\text{B-M}}$ (e^2fm^4)	$B(E2)_{ex.}$[13](e^2fm^4)	β_{2th} su3 int.	β_{2ex} [13]
18	1.50	1.88	160.5 (34)	243 (16)	0.5368 (53)	0.661 (21)
20	1.50	1.63	346 (4)	333 (16)	0.7347 (13)	0.72 (18)
22	1.50	1.27	283.7 (24)	229.8 (42)	0.6243 (10)	0.561 (51)
24	1.50	1.98	374.9 (161)	$143\left(\frac{+57}{-24}\right)$	0.6772 (63)	$0.418\left(\frac{+0.84}{-0.35}\right)$
26	1.50	2.01	281 (81)	155 (32)	0.5558 (34)	0.413 (43)
28	1.50	1.30	193.7 (42)	136 (32)	0.4384 (19)	0.367 (31)

Table 4
Reduced quadrupole deformation parameter for some nuclei (C, O, Ne, Mg, Ca).

Z, A	b	$B(E2)_{\text{exp.}}$[9](e^2fm^4)	$B(E2)_{th}$(e^2fm^4)	$\beta_{2ex.}$[9]	$\beta_{2th.}$	$Q_{ex.}$[9]($e\ fm^2$)	Q_{th}($e\ fm^2$)	$\delta_{ex.}$	$\delta_{th.}$
6,12	1.67	39.7(33)	75.7 (90)	0.582 (24)	0.8 (38)	20 (8)	27.5	0.36	0.5
8,18	1.751	45 (20)	38.9 (13)	0.355 (8)	0.33 (7)	21.29 (47)	19.7	0.24	0.22
10,20	1.774	340 (30)	300.6 (11)	0.727 (32)	0.68 (6)	58.4 (26)	54.9	0.50	0.47
12,26	1.83	305.6 (13)	334 (9)	0.482 (10)	0.5 (4)	55.4 (21)	57.9	0.35	0.36
20,42	1.95	420 (30)	95.8 (77)	0.247 (9)	0.117(53)	64.9 (23)	31	0.19	0.09

number of neutrons ($N = 8$). The theoretical calculation of the deformation parameter for isotope ^{18}Ne is close to the experimental data of the deformation parameter, while theoretical value of B(E2) underestimates the experimental data [14]. The theoretical calculation of the deformation parameter for isotope ^{20}Ne is close to the experimental data [14]. Table (1) showed that the deformation parameters are increased, reaching a maximum at $N = 10$ and then decreased. The analysis of the above calculations shows a strong contribution of *sd*-shell to the valence of two protons and two neutrons. The theoretical calculations of the deformation parameter for isotopes ^{22,24}Ne were close to the experimental data [14]. The calculations of isotopes ^{26,28}Ne were in agreement with the experimental data [14], as shown in Table (1).

Calculations of B(E2) are provided using the B-M effective charges [11] and *cw* interaction (CHUNG-WILDENTAL) [15] for isotopes ^{18,20,22,24,26,28}Ne in comparison with the experimental data [16] as shown in Table (2). The calculation of the deformation parameter β_{2th} from the B(E2) was adopted according to equation (7) and the results were compared with the experimental data [14]. The theoretical calculations for the B(E2) and $\beta_{2\text{th}}$ — in Table (2), showed a clear closeness between these calculations and the experimental data [14]. Also, the deformation parameter is increased, reaching a maximum of $N = 10$ and then decreased. The analysis of the above calculations showed a strong contribution of *sd* shell to the valence of two protons and two neutrons. The theoretical calculation of the deformation parameter for isotopes ^{18}Ne and ^{20}Ne agreed with the experimental error of the experimental data [14].

The theoretical calculations of the deformation parameter for isotopes ^{22,24,26,28}Ne were close to the experimental data [14].

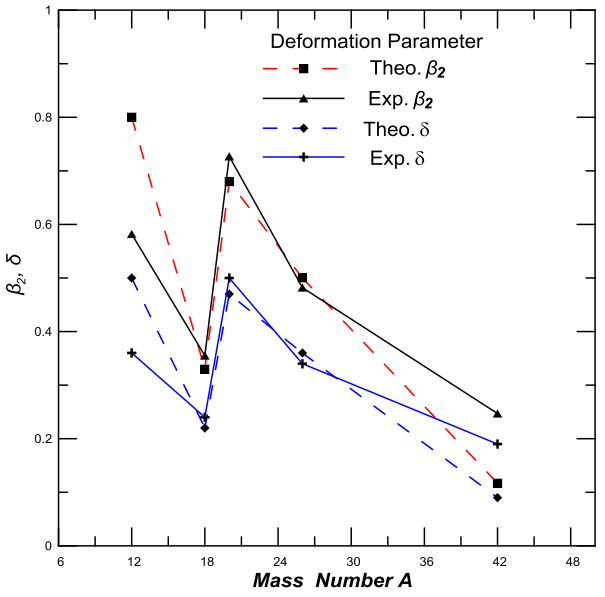

Figure (1). Quadrupole deformation parameter β_2 and δ for some nuclei (C, O, Ne, Mg, Ca).

The B(E2) was calculated from su3 interaction (Elliott J P) [16] for ^{18,20,22,24,26,28}Ne isotopes and adopted equation (7) to calculate the quadrupole deformation parameter, and the calculations were compared with the experimental data [14]. The theoretical calculation of the deformation parameter for the isotope ^{18}Ne agreed with the experimental error of the experimental data, and the isotope ^{20}Ne entirely agreed with the experimental data [14]. Table (3) showed that the quadrupole deformation parameter increased, reaching a maximum of $N = 10$ and then decreased. The analysis of the above calculations showed a strong contribution of *sd* shell to the valence of two protons and two neutrons. The theoretical calculations of the deformation parameter for isotope ^{22}Ne are close to the experimental data, and the theoretical calculation of the deformation parameter for isotope ^{24}Ne agreed with the experimental error of the experimental data. The calculations of the deformation parameter for the isotope ^{26,28}Ne were close to the experimental data [14], as demonstrated in Table (3).

Also, the quadrupole deformation is calculated from different model spaces such as the C nucleus for *p*-shell, the O, Ne and Mg nuclei for *sd* - shell and the Ca nucleus for *fp* shell using two methods. The first method from the reduced quadrupole transition B(E2) adopted equation (7) to calculate quadrupole deformation parameter (β_{2th}) where B(E2) was taken from Ref. [9]. The second method to calculate the quadrupole deformation parameter (δ_{th}) from the intrinsic quadrupole moment $Q^{'}$ adopted equation (8) where Q' was taken from Ref. [9], while the values of $\langle r^2 \rangle$ were calculated from equation (10). Table (4) and figure (1) showed the comparison of these calculations with the experimental data [9]. The calculations of δ_{th} and $\beta_{2\text{th}}$ agreed with the experimental data for the nuclei in *sd* shell, while calculations of $\beta_{2\text{th}}$ overestimated the experimental data for nuclei in *p*-shell and underestimated the nuclei in *fp*-shell. The theoretical calculations of deformation parameter δ were agreed with the experimental deformation parameter $\beta_{2\text{ex}}$ according to equation (11), where it was less than them by approximately 0.95 because it adopted the intrinsic quadrupole moment $Q^{'}$ which was taken from the experimental data [9].

4. Conclusion

In this study, the OBDM was calculated from each of the neon isotopes (^{18,20,22,24,26,28}Ne) using different interactions. Also, the OBDM was calculated from each of the assumed neon isotopes and using *sd*-model space. The transition rate B(E2) was calculated from some isotopes using the effective charges B-M for each isotope which represented the effect of polarization of the core. The quadrupole deformation parameters β_{2th} were calculated from some Ne isotopes ($A = 18$, 20, 22, 24, 26, 28) using three different interactions. The theoretical results of the β_{2th} were compared with the experimental data β_{2ex}. The quadrupole deformation parameter (β_{2th}, δ_{th}) was calculated by two methods, the B(E2) (the transition rate) method and $Q^{'}$ (the intrinsic quadrupole moment) method.

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