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OPTICALLY THICK RADIATING FREE CONVECTIVE MHD NANOFLUID FLOW OVER AN EXPONENTIALLY ACCELERATED PLATE

D.P. Bhatta
Siksha O Anusandhan Deemed to be University, debi.bhatta@gmail.com

S.R. Mishra
Siksha O Anusandhan Deemed to be University, satyaranjan_mshr@yahoo.co.in

J.K. Dash
Siksha O Anusandhan Deemed to be University, jkdash@gmail.com

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OPTICALLY THICK RADIATING FREE CONVECTIVE MHD NANOFLUID FLOW OVER AN EXPONENTIALLY ACCELERATED PLATE

Abstract

The present analysis investigates an unsteady conducting water-based nanofluid embedding with porous medium over an exponentially accelerated vertical plate. The plate is accelerated with moving ramped temperature. However, water is treated as the base fluid with Copper (Cu) and Titanium Oxide (TiO_2) as nanoparticles. Effects of thermal radiation, heat source, and radiation absorption are taken care of in the energy equation which may enhance the heat transfer properties of nanofluid. The crux of the investigation is to find the closed-form solution of nonlinear coupled partial differential equations. Laplace Transform technique is employed to solve these equations. The influence of the contributing parameters for the flow phenomena, in particular, thermal buoyancy parameter, thermal radiation, heat source/sink, and Prandtl number along with others are obtained and presented via graphs. Rate of shear stress and heat transfer coefficients for the significance of physical quantities of interest are also obtained and presented through graphs. Results and discussion section is devoted which elaborates on the behaviors of these, the emerging parameters. However, augmentation in the thermal profile is observed due to the heat sink as well as the thermal radiation parameters and nanoparticle volume fraction retards the velocity distribution due to the heavier density of the Cu nanoparticles.

Keywords

Nanofluids; Thermal buoyancy; Radiative heat energy; Porous medium; Laplace transformation.

1. Introduction

The homogeneous mixture of convectional base fluids i.e. water, ethylene glycol, oils, polymer solutions and other lubricants and very small-sized nanoparticles of diameter less than 100 nm is termed as nanofluid. The nanofluids have widespread applications in various engineering and industrial processes like the use of coolants for the nuclear reactors, microelectronics, automobiles, transformer coolant, domestic refrigerator, and cancer therapy, etc. due to their greater thermal efficiency in comparison to the base fluids. Choi [1] first initiated the work on nanofluid. He proposed that the thermal performance of base fluid may be increased by the dispersion of nanoparticles into the base fluid. Jang and Choi [2] reported the

Nomenclature

T	time[s]
T_w	wall temperature [K]
C_f	skin−friction coefficient
K	thermal conductivity [$Wm^{-1}k^{-1}$]
R	thermal radiation
Pr	Prandtl number
J	current density
Gr	thermal Grashof number
B_0	magnetic field strength
x,y	coordinate[m]
u,v,w	velocity components x,y *and* z direction respectively[ms^{-1}]
T	fluid temperature
T_∞	temperature away from the wall[k]
G	gravitational force[$m^2\ s^{-1}$]
Nu	Nusselt number
Q	heat generation/absorption
U_0	constant
Qw	wall heat flux[Ws^{-1}]
T	fluid temperature [K]
M	magnetic field parameter
Greek symbols	
∇p	pressure gradient
$k*$	mean absorption co-efficient[m^{-1}]
μ_f	fluid viscosity [$kgm^{-1}\ s^{-1}$]
β_T	thermal expansion coefficient
σ	electrical conductivity[$\Omega^{-1}m^{-1}$]
σ*	Stefan−Boltzmann constant[$kgs^{-3}K^{-4}$]
τ	heat capacity ratio
ρnf	nanofluid density [kgm^{-3}]
f	velocity profile
ϕ	volume fraction[mol m^{-3}]
θ	temperature profile
ρf	fluid density [kgm^{-3}]
ρcp	heat capacity[$Jm^{-3}K^{-1}$]
νf	kinematic viscosity[$m^2 s^{-1}$]
η,τ	similarity variable
ρs	nanoparticles density [kgm^{-3}]

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effects of various parameters on the thermal conductivity of nanofluid and Kang Ki and Choi [3] analyzed the convective instability and heat transfer characteristics of nanofluids. Several researchers have discussed the flows of nanofluid and its applications, through different geometries [4−17].

Magnetohydrodynamics (MHD) is the study of the interaction between fluid conductors of electricity and magnetic fields. The magnetohydrodynamic (MHD) nanofluids possess both the magnetic and liquid properties and have a wide range of applications in wavelength filters used in the magneto-optical system, optical switches, ink float separation, nonlinear optical materials, optical gratings, drug delivery for cancer treatment, etc. The separation of metals bearing a high melting point from non-metallic substances is another important application of MHD. The existence of electro magnetohydrodynamic waves was introduced by Alfven [18]. Due to vast applications of MHD, several researchers, namely Chamkha and Aly [19], Hamad and Pop [20] and Hayat et al. [21], have investigated on MHD flow of radiative nanofluids through various physical configurations. The ramped temperature boundary conditions on the free convective time-dependent MHD flow over a vertical plate was discussed by Seth et al. [22−24] and Nandkeolyar et al. [25] gave the solution of magnetohydrodynamic flow of fluid in conjunction with dust particles past an impulsively moving vertical plate considering the ramped wall temperature. Rashidi et al. [26] analyzed the influence of free convection on the magnetohydrodynamic fluid flow, heat and solutal transfer over a permeable vertical stretching surface. Rate of shear stress along with the influence of the thermal phenomena on the time-dependent flow of MHD viscous fluid in a porous medium was analyzed by Ali et al. [27]. Khalid et al. [28] have studied unsteady free convective MHD Casson fluid flow over an oscillating vertical plate embedding with a porous medium.

In thermal radiation, heat energy is emitted as electromagnetic waves in any directions from a radiating surface. For the creation of high temperature, the use of thermal radiation is vital. Various engineering processes such as nuclear power plants, gas turbines, gas-cooled nuclear reactors, various propulsion devices for aircraft, satellites, rocket combustion, space vehicles, missile's reentry, etc. Take place at high-temperature levels where thermal radiation has a decisive effect. Many researchers have taken an interest to study on the hydromagnetic convective flow of nanofluids for the influence of radiation under different geometries and configurations. Das et al. [29] studied the magneto-hydrodynamic flow of nanofluids in conjunction with the influence of radiation past on a moving porous plate. Haq et al. [30] analyzed the thermal radiation and slip effects on MHD stagnation point flow of nanofluid over a stretching sheet. Free convection as well as the effect of radiation on the MHD nanofluid past a stretching surface has been discussed by Rashidi et al. [31]. Moreover, the unsteady free convective flow of conducting fluid for the effect of ramped temperature boundary conditions over a moving vertical plate has been studied by Seth and Sarkar [32]. Khan et al. [33] investigated the MHD flow of Carreau nanofluid past a convectively heated surface in the presence of thermal radiation. Rehman and Eltayeb [34] considered the convective boundary condition in their study of heat transfer in a hydromagnetic radiative nanofluid over a nonlinear stretching sheet. Narayan et al. [35] studied the Hall current effects on MHD flow with free convection past a porous plate. Seddeek [36] analyzed the impacts of radiation and variable viscosity on MHD free convection flow over a semi-infinite flat plate. However, a methodology is employed by Zangooee et al. [37] to analyze the flow of a conducting nanofluid between two rotating disks. Derakhshan et al. [38] investigated the flow of MHD nanofluid through two parallel disks and they have used Adomian Ganji Method (AGM) to solve the transformed nonlinear equations. Further, Ganji and his coworkers ([39−45]) studied the flow behavior of several nanofluids for the variation of pertinent physical parameters in different geometries.

Based on the aforesaid knowledge, it is our objective to discuss an electrically conducting time-dependent nanofluid flow over an exponentially vertical heated plate embedded in a porous medium. In addition to that, free convection occurs due to the interference of the thermal buoyancy effect and the energy transfer equation is enhanced with the inclusion of heat sink and thermal radiation. Based on the physical situation we have used analytical solution, in particular, Laplace Transformation technique. Though the differential equations are partial due to the unsteadiness it is worthy to use Laplace transform technique. Withdrawing considered parameters from the present discussion our result validates with the earlier study of Hussain et al. [46]. Further, the behavior of several parameters is obtained and discussed.

2. Problem analysis and its formalism

Optically thick radiating water-based an electrically conducting nanofluid past an exponentially accelerated ramped wall temperature embedded with porous medium, placed vertically upward is taken in to account in the present investigation. The flow takes place along x^*- axis and it's usual to know that its transverse direction is y^*- axis. It is assume that for $t^* \leq 0$the fluid is at rest i.e. no flow occurs. The plate velocity is considered to be accelerated exponentially i.e. $U_0 e^{a^* t^*}$along the flow direction and the plate temperature assumed to be uniform i.e. T_w^*. Transverse magnetic field of strength $B_0$is applied normal to the flow (Fig. 1). The vector form of the governing equations of nanofluid is:

$$\nabla \cdot \mathbf{U} = \mathbf{0} \tag{1}$$

$$\rho_{nf}\frac{\partial \mathbf{U}}{\partial t} = -\nabla p + \mu_{nf}\nabla^2\mathbf{U} + \mathbf{J}\times\mathbf{B} + \mathbf{F}, \tag{2}$$

$$\left(\rho c_p\right)_{nf}\frac{\partial \mathbf{T}}{\partial t} = -\nabla\cdot\mathbf{q}, \tag{3}$$

where $\mathbf{U} = (u,v,w), q = -k_{nf}\nabla\mathbf{T}$ the heat flux, and $\mathbf{J} = \sigma_{nf}(\mathbf{E} + \mathbf{U}\times\mathbf{B})$ the current density.

The radiative heat flux based on Rosseland approximations (q_r) is also considered. Here, it is consider that the pressure gradient is isolated. Metal such as copper (Cu) and metal oxide i.e. TiO_2 nanoparticles are submerged with the base fluid water is considered as nanofluid. Following Das and Jana [29] the governing equations for the flow phenomena along with their surface conditions are

$$\rho_{nf}\frac{\partial u^*}{\partial t^*} = \mu_{nf}\frac{\partial^2 u^*}{\partial y^{*2}} - \sigma_{nf}B_0^2 u^* - \frac{\mu_{nf}}{K}u^* + g(\rho\beta_T)_{nf}(T^* - T_\infty{}^*) \tag{4}$$

$$\left(\rho c_p\right)_{nf}\frac{\partial T^*}{\partial t^*} = k_{nf}\frac{\partial^2 T^*}{\partial y^{*2}} - \frac{\partial q_r}{\partial y^*} + Q_0(T^* - T_\infty{}^*) \tag{5}$$

$$\left.\begin{array}{lll} t\leq 0; & u^* = 0, T^* = T_\infty{}^*, & \forall y\geq 0, \\ t>0: & u^* = U_0 e^{a^*t^*}, T^* = T_w{}^*, & at\, y = 0, \\ t>0: & u\to 0, T^*\to T_\infty{}^* & as\, y\to\infty \end{array}\right\} \tag{6}$$

Now, imposing the Rosseland approximation (Hayat et al. [25]),

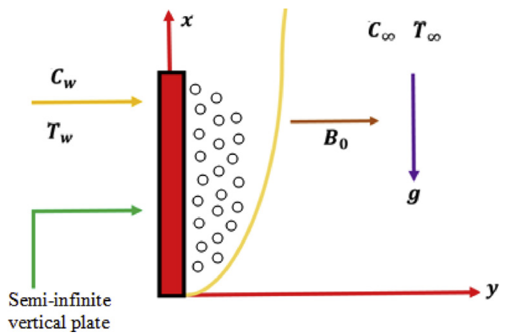

Fig. 1. Geometrical configuration.

$$q_r = -\frac{4\sigma^*}{k^*}\frac{\partial T^{*4}}{\partial y^*} \tag{7}$$

Further simplifying T^{*4} using Taylor series expansion about $T_\infty{}^*$ and considering only linear term we get

$$T^{*4} \approx 4T_\infty^{*3}T^* - 3T_\infty^{*4} \tag{8}$$

Employing above expression Eq. (6) is of the form;

$$\left(\rho c_p\right)_{nf}\frac{\partial T^*}{\partial t^*} = k_{nf}\frac{\partial^2 T^*}{\partial y^{*2}} + \frac{16T_\infty^{*3}\sigma^*}{3k^*}\frac{\partial^2 T^*}{\partial y^{*2}} + Q_0(T^* - T_\infty{}^*) \tag{9}$$

Following Das and Jana [29] the physical properties of nanofluid are given by following

$$\rho_{nf} = (1-\phi)\rho_f + \phi\rho_s, \mu_{nf} = \frac{\mu_f}{(1-\phi)^{2.5}},$$

$$\left(\rho c_p\right)\left(\rho c_p\right)_{nf} = (1-\phi)\left(\rho c_p\right)_f + \phi\left(\rho c_p\right)_s,$$

$$(\rho\beta_t)(\rho\beta_t)_{nf} = (1-\phi)(\rho\beta_t)_f + \phi(\rho\beta_t)_s,$$

$$\sigma_{nf} = \sigma_f\left[1 + \frac{3(\sigma_s/\sigma_f - 1)\phi}{(\sigma_s/\sigma_f + 2) - (\sigma_s/\sigma_f - 1)\phi}\right],$$

$$k_{nf} = k_f\left\{\frac{k_s + 2k_f - 2\phi(k_f - k_s)}{k_s + 2k_f + \phi(k_f - k_s)}\right\}$$

With the help of following non-dimensional quantities

$$y = \frac{y^*}{U_0 t_0}, t = \frac{t^*}{t_0}, u = \frac{u^*}{U_0}, T = \frac{T^* - T_\infty{}^*}{T_w{}^* - T_\infty{}^*}$$

and substituting in equations (4), (6) and (9), we get;

$$\frac{\partial u}{\partial t} = r_1\frac{\partial^2 u}{\partial y^2} + r_2 GrT - \left(r_3 M + \frac{1}{Kp}\right)u, \tag{10}$$

$$\frac{\partial T}{\partial t} = r_4\frac{\partial^2 T}{\partial y^2} + r_5 T, \tag{11}$$

$$\left.\begin{array}{ll} t\leq 0: & u = 0, \quad T = 0, \quad \forall y\geq 0, \\ t>0: & \left\{\begin{array}{lll} u = e^{at}, & T = 1, & at\ y\geq 0, \\ u\to 0, & T\to 0 & as\ y\to\infty \end{array}\right. \end{array}\right\} \tag{12}$$

$$M = \frac{\sigma_f B z_0{}^2 \nu_f}{U_0\rho_f}, Gr = \frac{g\beta_{Tf}(T_w - T_\infty)\nu_f}{U_0{}^3}, \Pr = \frac{\left(\mu c_p\right)_f}{k_f},$$

$$R = \frac{4\sigma^* T_\infty^3}{k^* k_f}, Q = \frac{Q\nu_f}{U_0^2\left(\rho c_p\right)_f}, Kp = \frac{K\nu_f}{U_0^2}.$$

3. Solution methodology

Solution of these equations 10 and 11 with the given boundary conditions (12) is obtained analytically employing Laplace transformation technique. By applying Laplace transform on both side of equations 10 and 11 we get,

$$r_1\frac{d^2\overline{u}}{dy^2}-\left(r_3M+\frac{r_1}{Kp}+s\right)\overline{u}+r_2Gr\overline{T}=0, \tag{13}$$

$$r_4\frac{d^2\overline{T}}{dy^2}+(r_5-s)\overline{T}=0, \tag{14}$$

$$\left.\begin{array}{llll}\overline{u}=0, & \overline{T}=0, & \forall y\geq 0, t=0 & \\ \overline{u}=\dfrac{1}{s-a}, & \overline{T}=\dfrac{1}{s}, & at & y=0, \\ \overline{u}\to 0, & \overline{T}\to 0, & as & y\to\infty\end{array}\right\} \tag{15}$$

Now the ordinary differential equations 13 and 14 and boundary conditions presented in (15) are solved and we get the following.

$$\overline{T}(s,y)=\frac{1}{s}e^{-\sqrt{\frac{1}{r_4}(s-r_5)}y}$$

$$T(t,y)=\frac{1}{2}\left\{e^{\sqrt{-\frac{r_5}{r_4}}y}erfc\left(\frac{y}{2\sqrt{r_4t}}+\sqrt{-r_5\,t}\right)+e^{-\sqrt{-\frac{r_5}{r_4}}y}erfc\left(\frac{y}{2\sqrt{r_4t}}-\sqrt{-r_5\,t}\right)\right\}$$

$$\overline{U}(s,y)=\left[\frac{1}{s-a}-\frac{r_8}{r_7r_9}\left(\frac{1}{s-r_9}-\frac{1}{s}\right)\right]e^{-\frac{1}{\sqrt{r_1}}\sqrt{s+M_1}y}+\frac{r_8}{r_7r_9}\left(\frac{1}{s-r_9}-\frac{1}{s}\right)e^{-\frac{1}{\sqrt{r_4}}\sqrt{s-r_5}y}$$

$$u(t,y)=\frac{1}{2}\left[\begin{array}{l} e^{at}\left\{e^{\sqrt{\frac{1}{r_1}(M_1+a)}y}erfc\left(\frac{y}{2\sqrt{r_1t}}+\sqrt{(M_1+a)\,t}\right)+e^{-\sqrt{\frac{1}{r_1}(M_1+a)}y}erfc\left(\frac{y}{2\sqrt{r_1t}}-\sqrt{(M_1+a)\,t}\right)\right\} \\ -\frac{r_8}{r_7r_9}e^{r_9t}\left\{e^{\sqrt{\frac{1}{r_1}(M_1+r_9)}y}erfc\left(\frac{y}{2\sqrt{r_1t}}+\sqrt{(M_1+r_9)\,t}\right)+e^{-\sqrt{\frac{1}{r_1}(M_1+r_9)}y}erfc\left(\frac{y}{2\sqrt{r_1t}}-\sqrt{(M_1+r_9)\,t}\right)\right\} \\ +\frac{r_8}{r_7r_9}\left\{e^{\sqrt{\frac{M_1}{r_1}}y}erfc\left(\frac{y}{2\sqrt{r_1t}}+\sqrt{M_1\,t}\right)+e^{-\sqrt{\frac{M_1}{r_1}}y}erfc\left(\frac{y}{2\sqrt{r_1t}}-\sqrt{M_1\,t}\right)\right\}+ \\ \frac{r_8}{r_7r_9}e^{r_9t}\left\{e^{\sqrt{\frac{(r_9-r_5)}{r_4}}y}erfc\left(\frac{y}{2\sqrt{r_4t}}+\sqrt{(r_9-r_5)\,t}\right)+e^{-\sqrt{\frac{(r_9-r_5)}{r_4}}y}erfc\left(\frac{y}{2\sqrt{r_4t}}-\sqrt{(r_9-r_5)\,t}\right)\right\}- \\ \frac{r_8}{r_7r_9}\left\{e^{\sqrt{\frac{-r_5}{r_4}}y}erfc\left(\frac{y}{2\sqrt{r_4t}}+\sqrt{-r_5\,t}\right)+e^{-\sqrt{\frac{-r_5}{r_4}}y}erfc\left(\frac{y}{2\sqrt{r_4t}}-\sqrt{-r_5\,t}\right)\right\} \end{array}\right]$$

Table-1
Physical properties of nanoparticles and base fluid.

Nanoparticle/Base Fluid	$\rho(Kg/m^3)$	$C_p(J/KgK)$	$k(W/mK)$	$\beta \times 10^5(K^{-1})$	$\sigma(S/m)$
Cu	8933	385	401	1.67	5.96×10^6
TiO_2	4250	686.2	8.9583	0.90	2.5×10^6
Water	997.1	4179	0.613	21	5.5×10^6

Table-2
Validation for the skin friction coefficient in particular case of Pr = 0.71 with % error.

M	ϕ	R	t	Present	Hussain et al. [46]	% Error
6	0	1	0.5	0.76932	0.76661	0.3522

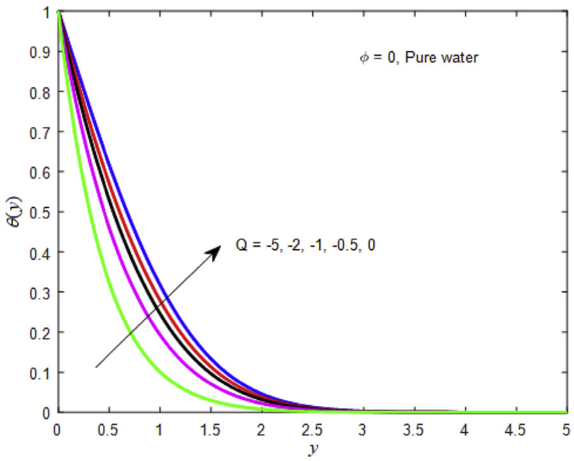

Fig. 2. Variation of Q for pure water and R = 0.

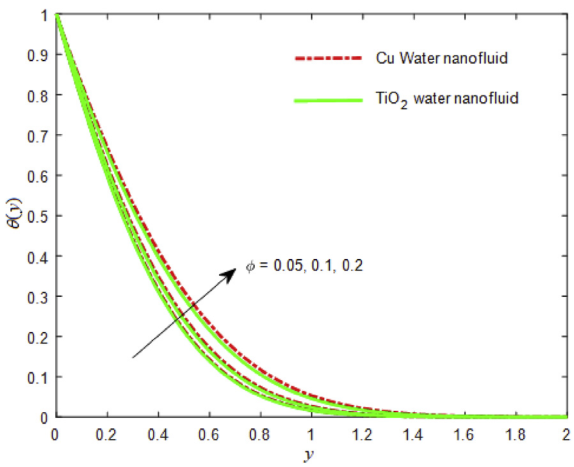

Fig. 3. Variation of ϕ for nanofluids and R = 0.

All the constants reported in appendix.

4. Results and discussion

Unsteady flow of *Cu* and *Tio_2*-water nanofluids past an optically thick moving vertical exponentially accelerated plate are investigated in the present problem. Conducting fluid embedding with a porous

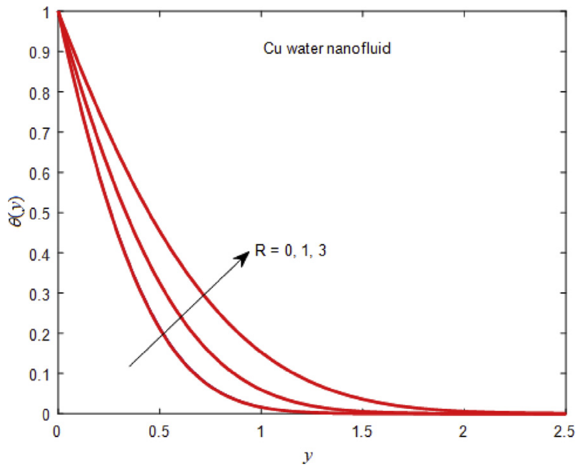

Fig. 4. Variation of R for Cu water nanofluid.

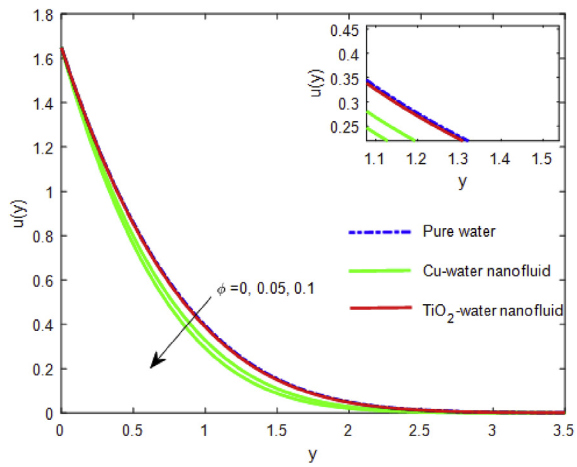

Fig. 5. Variation of ϕ on velocity profiles.

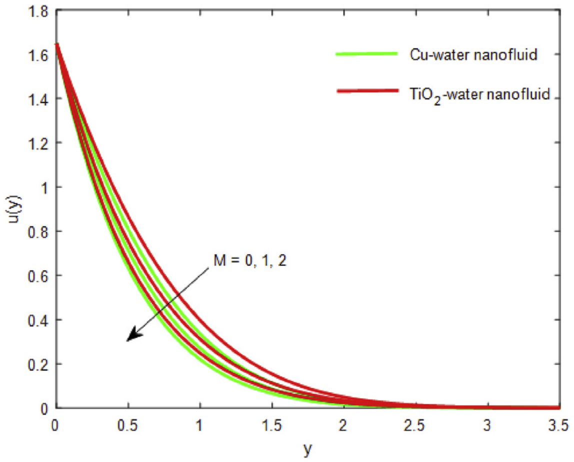

Fig. 6. Variation of M on velocity profiles.

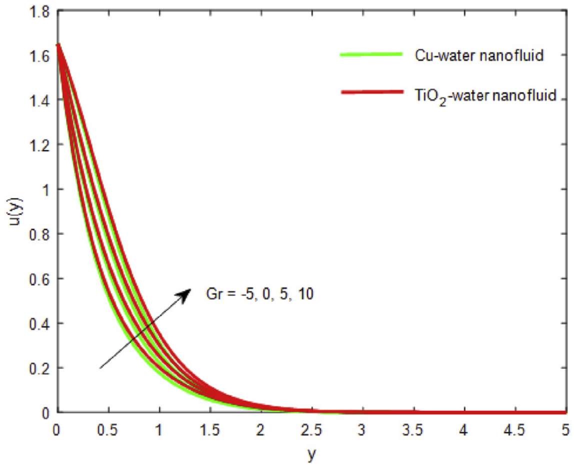

Fig. 8. Variation of *Gr* on velocity profiles.

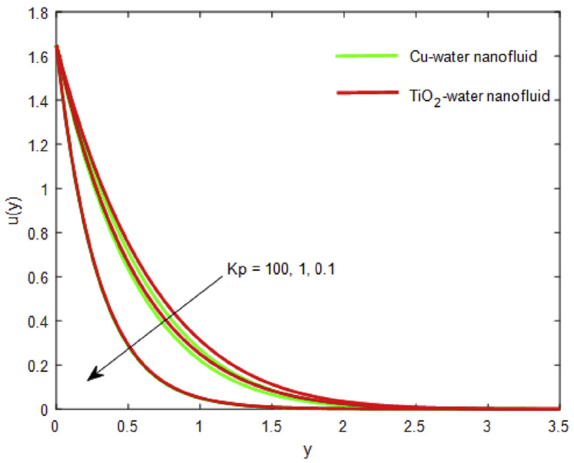

Fig. 7. Variation of Kp on velocity profiles.

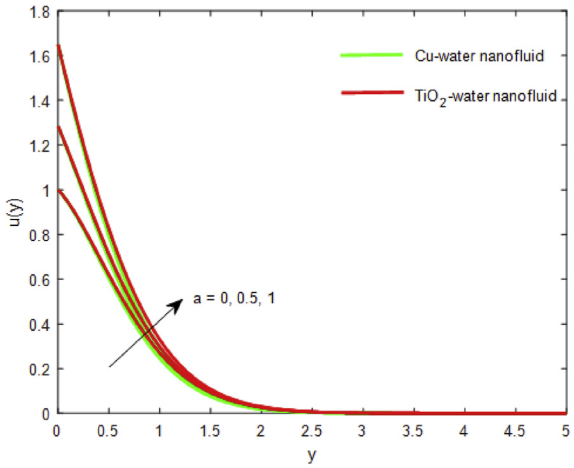

Fig. 9. Variation of *a* on velocity profiles.

medium in the presence of radiative heat energy and internal heat generation/absorption parameter. Analytical solution for the transformed non-dimensional nonlinear differential equations is employed using the Laplace transformation technique. The influences of various characterizing parameters on the flow phenomena with the effective properties of nanoparticles and kerosene were considered and displayed via graphs. Table-1 presents the thermophysical properties of nanoparticles along with the base fluid. Moreover, the validation and conformity of the said outcomes and solution methodology with the results of Hussain et al. [46] is obtained and displayed in Table-2 in a particular case. From the computation of percentage error, it is found that the results are in excellent agreement. In the entire discussion, we have considered kerosene as the base fluid with copper (Cu) and Titanium oxide (Tio_2) nanoparticles and the quantities of engineering interest are shown in Figs. 2–13.

4.1. Temperature distributions

Fig. 2 displays the comparison plot for the temperature distribution in the absence of thermal radiation and the presence/absence of heat sink parameter for $\phi = 0$ (pure fluid case). In the case of pure fluid ($\phi = 0$) (marked in blue) and for $t = 0.1$ and in the absence of heat sink parameter ($Q = 0$), the present result validates with the work of Hussain et al. [46]. However, it is observed that an increase in heat sink the fluid temperature increases significantly. It is due to the fact that in the case of viscous fluid presence of the sink produces heat energy near the boundary layer and

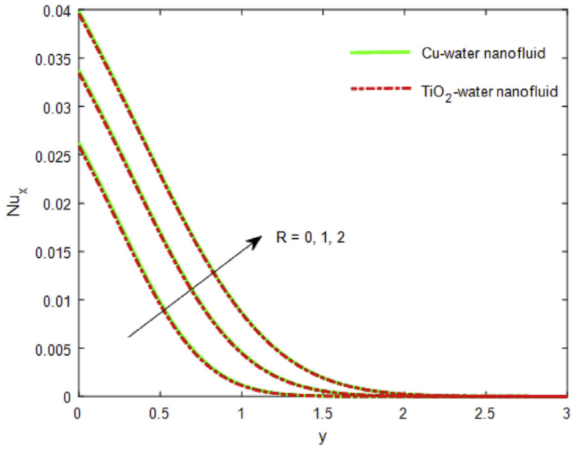

Fig. 10. Variation of R on rate of heat transfer.

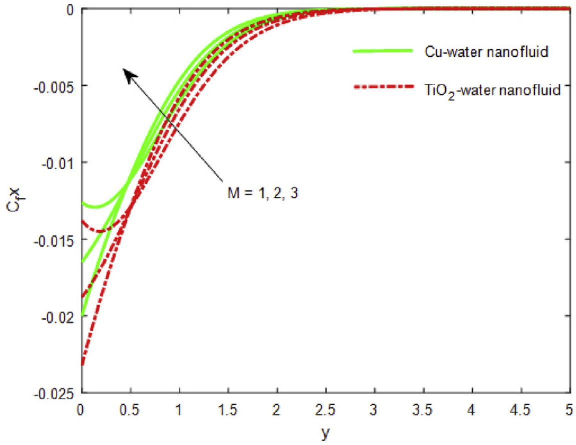

Fig. 12. Variation of M on rate of shear stress.

the energy stored up which resulted in enhancing the fluid temperature. Fig. 3 describes the effects of nanoparticle volume fraction in the absence of thermal radiation on the temperature profiles of both *Cu*-and TiO_2-water nanofluids. It is a clear indication from the figure that thermal boundary layer thickness becomes thinner with low values of volume fraction ($\phi = 0.005$) whereas with an increasing volume fraction ($\phi = 0.1\%–0.2\%$) the thickness of the thermal boundary layer increases resulted in the fluid temperature increases. In comparison to both the nanofluids, it is seen that the Cu-water nanofluid is favorable to enhance the fluid temperature and concluded that *Cu* is the good conductor of heat which enhances the thermal boundary layer thickness at all points in its boundary layer. Fig. 4 shows the variation of thermal radiation on the Cu-water nanofluid temperature profiles for the volume fraction $\phi = 0.05$. In the absence of thermal radiation ($R = 0$) the fluid attains its minimum value which coincides with the result of Hussain et al. [46] and further, it is marked that nanofluid temperature enhances with an increase in radiation. "Thermal radiation is the electromagnetic transfer of energy from a hotter surface to colder surface and thus represent heat transfer without physical medium" [28]. Therefore, the increase in thermal radiation increases the *Cu*-water nanofluid temperature.

4.2. Velocity distributions

Fig. 5 portrays the validation of the present work with the work of Hussain et al. [46] in the absence of magnetic parameter M ($M = 0$), porous matrix Kp($Kp = 100$), heat sink parameter Q ($Q = 0$) in case of both pure fluid ($\phi = 0$) and the nanofluids. Also, in the variation of nanoparticle volume fraction on the velocity profiles is

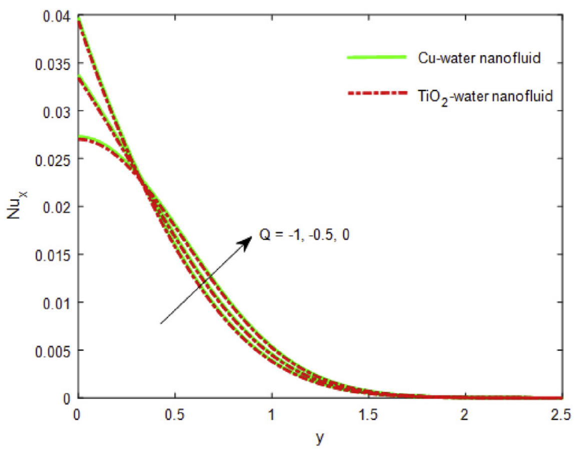

Fig. 11. Variation of Q on rate of heat transfer.

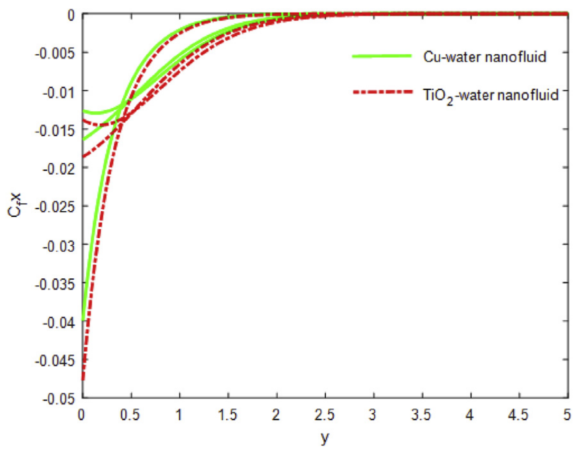

Fig. 13. Variation of Kp on rate of shear stress.

exhibited for both Cu-water and TiO_2-water-based nanofluids. It is observed that in the case of pure fluid, the profile attains its maximum value which is far away from the plate. Further, the increase in volume fraction the velocity decreases for which the velocity boundary layer thickness decreases at all points within the flow domain. It is interesting to observe that due to the heavy density of the Cu-nanoparticles in comparison to TiO_2 nanoparticles the fluid velocity retards and the boundary layer thickness decreases. From Fig. 6 it is noteworthy that, retardation in the momentum of the *Cu*-water nanofluid is marked for increasing magnetic parameter ($M \neq 0$) and the absence of porous matrix ($Kp = 100$) in the velocity boundary layer. The interaction of the magnetic parameter presents a usual phenomenon of in the fluid motion and the fact is, "the inclusion of the magnetic parameter produces Lorentz force which is a resistive force has a significant role to retards the velocity profile". The retardation of the profiles within the region $y < 1.5$ is rapid whereas afterward it decreases smoothly to meet the boundary condition. The influence of porous matrix in the presence/absence of magnetic parameter for both Cu and TiO_2 water nanofluid is displayed in Fig. 7. The absence of both the parameter is described earlier in Fig. 5 which coincides with the earlier work. Nanofluid velocity profiles decelerate in the presence of a porous matrix for both the presence/absence of the magnetic parameter. As magnetic parameter, porosity causes a resistive force that resists the velocity significantly. It is clear to mark that TiO_2-water-based nanofluid has a more viscous effect than that of *Cu*-Water nanofluid. So, that the profiles for Cu-water nanofluid are closer towards the plate. The fact is due to the heavier density of Cu nanoparticles than TiO_2 nanoparticles. The influence of the thermal buoyancy parameter *Gr* on both Cu-water and TiO_2-water nanofluid velocity profile is shown in Fig. 8. Here, $Gr > 0$ indicates cooling of the plate and $Gr < 0$ represents heating of the plate. It is observed that cooling of the plate favors in to enhance the fluid temperature whereas the heating retards it significantly. Further, TiO_2-water nanofluid produces maximum energy at all points in the boundary layer than the Cu-water nanofluid. Fig. 9 describes the variation of coefficient of exponent *a* on the velocity profiles of both Cu-and TiO_2 water nanofluid. It is clear to observe from the mathematical expression that as the exponent increases the nature of exponential function also grows rapidly. That is also seen from the figure that with an increasing value of coefficient *a* the profile increases throughout the domain for both the nanofluids. In comparison to both nanofluids, it is observed that the TiO_2 nanofluid is more pronounced than that of Cu-kerosene nanofluid. The fact is that, the density of Cu is more than that of TiO_2 for which the velocity boundary layer thickness retards more in the case of Cu.

4.3. Results of engineering interest

Finally, Figs. 10−13 display the engineering interests via local Nusselt number and rate of shear stress for various pertinent physical parameters. In Fig. 10, it is seen that the rate of heat transfer increases significantly with an increase in the thermal radiation parameter. The comparison between Cu and TiO_2 nanofluids is insignificant. Sharp fall in the profile is marked ear the plate surface within a region $\eta < 0.75$ and afterward the effect is smooth to meet the boundary conditions. Again Fig. 11 depicts the variation of the heat sink on the rate of heat transfer profiles of both the nanofluids undertaken for the study. Distinct character is marked in different regions for different values of the heat sink parameter. Here, $Q = 0$ represents the no additional heat source is a comparison to earlier study and the behavior of heat sink is observed for $Q < 0$. From the point of inflection at $\eta = 0.25$ the deviation in the profile is marked. In the region $\eta = 0.25$ heat sink retards the rate of heat transfer whereas in the second region i.e. $\eta = 0.25$ effect is reversed i.e. Rate of shear stress for various values of the magnetic and porous parameter is exhibited in Figs. 12 and 13 respectively. The hike in the profile from the negative region is marked in both the profiles. It is also observed that both the profile have a tendency to enhance the rate of shear stress with increasing magnetic and porous matrix. However, in comparison to the nanofluids, it is marked that, Cu-kerosene nanofluid produces a greater impact that TiO_2-kerosene nanofluid in the presence of both the parameters. Moreover, it is concluded that Cu nanoparticle is more useful for the enhancement in the heat transfer properties of conducting fluid.

5. Conclusion

Free convective flow of an electrically conducting nanofluid over a heated vertical plate is obtained in the present paper. An analytical approach such as Laplace Transform technique is used for the solution of the present model to discuss the behavior of various physical parameters on the flow phenomena. Presence of thermal radiation and heat sink parameter also enhance the study as well. From the aforesaid discussion, few major findings are laid down here.

- In particular situation present result validates with earlier established results.
- Heat sink favors in to enhance the fluid temperature in the boundary layer in the case of Cu-kerosene nanofluid.
- Due to the heavier density of Cu nanoparticles, an increase in nanoparticle volume fraction retards the velocity profiles.
- Thermal radiation is beneficial to hike the nanofluid temperature since *Cu*is a good conductor of heat.
- Radiation augmented with heat sink increases the heat transfer rate.
- Hike in the rate of shear stress is marked for both the resistive forces such as the magnetic parameter and the porous matrix.

Last but not least the present study reveals a wide range of applications to the society and warrants further study on nanofluid and a lot of scopes are there in future to study the further part of the model using various thermo-physical properties of nanofluids.

Appendix.

$$x_1=(1-\phi)+\phi\frac{\rho_s}{\rho_f}, x_2=(1-\phi)+\phi\frac{(\rho\beta_T)_s}{(\rho\beta_T)_f},$$

$$x_3=\left[1+\frac{3(\sigma-1)\phi}{\sigma+2-(\sigma-1)\phi}\right],$$

$$\sigma=\frac{\sigma_s}{\sigma_f},\ x_4=(1-\phi)+\phi\frac{(\rho c_p)_s}{(\rho c_p)_f},$$

$$x_5=\frac{k_{nf}}{k_f}=\left[\frac{k_s+2k_f-2k\phi(k_f-k_s)}{k_s+2k_f+\phi(k_f-k_s)}\right],$$

$$r_1=\frac{1}{(1-\phi)^{2.5}x_1},\ r_2=\frac{x_2}{x_1}, aa$$

$$r_3=\frac{x_3}{x_1},\ r_4=\frac{1}{x_4\mathrm{Pr}}\left\{x_5+\frac{3}{4}R\right\},\ r_5=\frac{Q}{x_4},$$

$$r_6=-\frac{r_2}{r_1}Gr, r_7=r_1-r_4,\ r_8=r_1r_4r_6,\ r_9=\frac{r_1r_5+r_4M_1}{r_1-r_4},$$

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