Karbala International Journal of Modern Science

Volume 6 | Issue 3 Article 12

Numerical approach to non-Darcy mixed convective flow of non-Newtonian fluid on a vertical surface with varying surface temperature and heat source

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Recommended Citation
Baitharu, Ajaya Prasad; Sahoo, Sachidananda; and Dash, Gauranga Charan (2020) "Numerical approach to non-Darcy mixed convective flow of non-Newtonian fluid on a vertical surface with varying surface temperature and heat source," *Karbala International Journal of Modern Science*: Vol. 6 : Iss. 3 , Article 12.
Available at: https://doi.org/10.33640/2405-609X.1753

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Abstract

An analysis is performed on non-Darcy mixed convective flow of non-Newtonian fluid past a vertical surface in the presence of volumetric heat source originated by some electromechanical or other devices. Further, the vertical bounding surface is subjected to power law variation of wall temperature, but the numerical solution is obtained for isothermal case. In the present non-Darcy flow model, effects of high flow rate give rise to inertia force. The inertia force in conjunction with volumetric heat source/sink is considered in the present analysis. The Runge-Kutta method of fourth order with shooting technique has been applied to obtain the numerical solution. To avoid mathematical impasse for applying R-K method we have considered isothermal wall condition. The results of major interest include velocity as well as temperature profiles and the local Nusselt number for some representative values of power-law indices. Most importantly, introduction of the co-ordinate and parametric transformation applied to governing equations, rarely reported in the existing literature, add to the knowledge front. Some important findings of the study are: Ergun number reduces the pseudoplastic fluid velocity boundary layer, a desirable outcome, but enhances the thermal boundary layer whereas, in case of Newtonian and dilatant fluid, the effect is not so significant. An increase in all the flow and heat transfer parameters leads to decelerate the surface cooling from pseudoplasticity to dilatancy through Newtonian; thus the present model slows down the surface cooling and decreases the skin friction in the presence of heat source for dilatant fluid.

Keywords

Mixed convection, Non-Darcy parameter (Ergun number), Saturated porous medium, Non-Newtonian fluid, Vertical surface

Cover Page Footnote

The authors gratefully acknowledge the referees for their constructive comments and valuable suggestions.

1. Introduction

Problems on fluid flow through porous media and heat transfer are not only the interests of Mathematicians but also of Chemical Engineers. The petroleum engineers working on miscible displacement process and the civil engineers dealing with the salt-water encroachment of coastal aquifers contribute to the areas of the present problem. In some practical situations, free and forced convections arise simultaneously. This particular case for a large Grashof number corresponds to extraction of crude oil. The mixed convection over a vertical plate in a porous medium is closely related to building up pressure gradients while

Nomenclature

x,y	coordinate axes
u,v	non-Darcian velocity components along x- and y-directions respectively
T	temperature of the fluid in the boundary layer
T_w	surface temperature
T_∞	temperature of the ambient fluid
U_∞	free stream velocity
x	distance from leading edge
Ra_x	local Rayleigh number
Ra_d	Rayleigh number based on the pore diameter
Pe_x	local Peclet number
Pe_d	Peclet number based on the pore diameter
Er	Ergun number based on the pore diameter
K^*	inertial coefficient in the Ergun equation
K_p	permeability
Q	rate of heat generation
Pr	Prandtl number
S	heat source parameter
n	viscosity index
g	acceleration due to gravity
d	pore diameter
A	a constant

Greek symbols

ρ	density of the fluid
ρ_∞	density of the ambient fluid
λ	power law exponent
μ	viscosity
v	kinematic viscosity
α	thermal diffusivity
β	volumetric coefficient of expansion
ξ	non-similarity mixed convection parameter
η	similarity variable
τ_w	shear stress at the wall

Subscripts

w	condition on the wall
∞	free stream region

https://doi.org/10.33640/2405-609X.1753

withdrawing or injecting geothermal fluids to the reservoirs. Further, due to the temperature difference between the plate and the flowing fluid, a buoyancy force is induced which generates a body force acting per unit volume besides the pressure gradient, an inherent force built up due to the motion of the fluid. The problem of mixed convection flow past a vertical surface in a porous medium saturated with a power-law is investigated by Gorla and Kumari [1]. Mahdy [2] studied the melting effect on the mixed convective heat transfer from a vertical surface embedded in a non-Newtonian fluid-saturated porous media. Chen et al. [3] have investigated non-Darcian flow phenomena on mixed convective transport for different flow models and concluded that non-Darcian and thermal dispersion affect significantly the velocity, temperature and heat transfer rates from a vertical surface. EL-Kabeir et al. [4] have considered unsteady, laminar, heat and mass transfer MHD mixed convective boundary-layer flow of electrically conducting fluid over an impulsively stretched vertical surface in an unbounded quiescent fluid with aiding the external flow and melting effects. Chamkha et al. [5] have considered the MHD mixed convective Nanofluid flow past a stretching permeable surface and pointed out the effects of buoyancy ratio and thermophoresis parameter using an iterative tri-diagonal implicit finite difference method. Chamkha et al. [6] have contributed to nanofluid flow over an isothermal vertical wedge embedded in a porous medium in the presence of thermal radiation. A numerical study has been carried out on solutal dispersion on heat and mass transfer in non-Darcy fluid flow over a vertical surface by Hemalata et al. [7]. They observed that the melting parameter enhances the velocity, but reduces the solutal concentration in the flow domain. Prasad et al. [8] have analyzed the problem of mixed non-Newtonian convection along a vertical plate considering melting and thermal dispersion-radiation effects for aiding and opposing external flows. Further, Kairi and Murthy [9] and Kairi and Ram Reddy [10] contributed to mixed convection flow in a non-conducting non-Newtonian fluid flow, but Barman et al. [11] studied the above problem with a cooling fluid under the influence of magnetic field on a Newtonian flow. Further, Rao et al. [12] studied the cross flow past a stretching surface with slip at the boundary. Kumari and Jayanthi [13] have studied a non-Darcy non-Newtonian mixed convection flow along a vertical plate. Sahoo et al. [14] analyzed two dimensional unsteady viscoelastic fluid (Walters model) flow through a porous medium ensurfaced by a vertical infinite porous plate. The bounding surface is subjected to fluid flow with a periodic velocity and the free stream velocity oscillates about a non-zero mean value in the main direction of the flow. Recently, Sahoo et al. [15] have considered a second grade, unsteady, electrically conducting, incompressible and rotating fluid flow through a channel with oscillating upper plate. Sharma et al. [16] have considered the buoyancy effects on MHD mixed convection of a radiating chemically reacting binary mixture past a vertical porous plate. They have remarked that variable viscosity enhances the flow and heat transfer rates at the surface. Athira et al. [17] have analyzed the effect of the induced magnetic field on a chemically reacting species across a vertical porous plate. They have observed that the non-linear convection has a destructive effect on thermal field and its layer thickness. The unsteady mixed MHD convection Blasius flow past a flat surface has been studied by Makinde et al. [18]. They have pointed out some interesting surface criteria on the flow and heat transfer phenomena. Sharma et al. [19] have analyzed MHD slip flow and heat transfer over an exponentially stretching sheet in a porous medium. They have presented a numerical solution depicting the effects of pertinent parameters on the slip flow.

In Rheology shear thinning contributes to non-Newtonian behavior of fluids that commensurate with the decreasing of viscosity under shear strain. The fluids having shear thinning property represent the most common type of fluids, i.e. pseudoplastic fluids abundantly used in industrial applications, for example, long chain polymers and blood etc. On the other hand, dilatant fluids are shear thickening fluids having a higher viscosity such as mixture of sand and water, and colloidal dispersion, etc. Flow of fluids through beds of granular solids is of frequent occurrences in industrial applications, particularly, in petroleum extraction processes and gas flow through crushed porous solids etc.

To obviate the limitations of Darcy model which neglects the boundary (viscous resistance) and inertia effects on fluid flow and heat transfer through high porosity media, the present model has been adopted for flow through high porosity media such as foam metals and fibrous media. The Ergun model [20] is suitable to study the inertia and boundary effects on flow along vertical plate. From the foregoing discussion, it is observed that convective flow through the saturated porous medium subjected to space varying temperature in the presence of the volumetric heat source of non-Newtonian fluid (Ergun model) has not been considered so far. The surge of interest for the present problem lies with the consideration of the simultaneous effects of fluid inertia force and boundary viscous resistance (non-Darcian flow model) with the help of Ergun flow model.

The non-Newtonian fluid model used in the present study takes care of the following aspects. The pressure losses caused by simultaneous kinetic and viscous-energy-losses accompanying the flow of fluids through columns packed with granular material, has been the subject of theoretical analysis. The present model considers simultaneously the effects of fluid inertia force and boundary viscous resistance (non-Darcian flow model) whereas Darcian model fails to accomplish. From the study of the previous researchers referred herein, it is seen that though the works are related to flow past vertical surface, still some differ in fluid models, physical ambience and geometrical configuration, but a few, such as Ergun [20] and Kumari and Jayanthi [13] have worked on the same model but without a heat source. The occurrence of heat source in the process-industries is a vital-consideration from the application point of view. Hence, the consideration of heat source is more realistic to deal with.

The novelty lies with the incorporation of a single non similarity parameter ξrepresenting the three states of convection such as free($\xi = 0$), forced($\xi = 1$) and mixed $(0 < \xi < 1)$as well as thermal transport phenomena with temperature dependent heat source. Further, the power-law wall temperature parameter $\lambda = 0$ represents the isothermal wall condition. Most importantly, the volumetric temperature-dependent heat source which is of frequent occurrence in industrial processes due to advent of electro-mechanical devices, either in generating or absorbing heat, has made the discussion more application oriented.

2. Formulation of the problem

Consider the steady two-dimensional flow over a semi-infinite vertical plate embedded in a saturated porous medium with surface temperature T_w and the ambient temperature T_∞.The undistorted uniform speed U_∞ engenders the flow. The coordinate system is presented in Fig. 1. We have the following assumptions:

(i) The flow considered is two dimensional, steady, laminar and incompressible.

(ii) The Boussinesq approximation is valid, which consists of two parts: (a) In the governing equations, all the variable property-effects are ignored except the density term appearing in the momentum equation. (b) The density difference term is approximated with a simplified equation of state that is

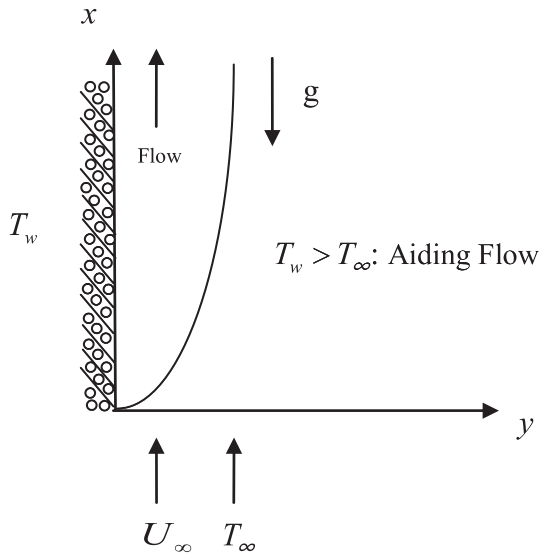


Fig. 1. Flow geometry.

$$\rho = \rho_\infty(1 - \beta(T - T_\infty)) \quad (1)$$

(iii) The thermo physical properties of the fluid are homogeneous and isotropic.

(iv) The surface temperature of the vertical heated plate varies in the power-law form

$$T_w(x) = T_\infty + Ax^\lambda \quad (2)$$

Under these assumptions, conservation equations for non-Darcy flow with boundary layer approximations following [13] are given by

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad (3)$$

$$\frac{\partial u^n}{\partial y} + \frac{K^*}{\nu}\frac{\partial u^{2n}}{\partial y} = \frac{K_p \rho g \beta}{\mu}\frac{\partial T}{\partial y} \quad (4)$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha\frac{\partial^2 T}{\partial y^2} + Q(T - T_\infty) \quad (5)$$

The boundary conditions are given by

$$\left.\begin{array}{l} y = 0 : v = 0, T = T_w = T_\infty + Ax^\lambda \\ y \to \infty : u \to U_\infty, T \to T_\infty \end{array}\right\} \quad (6)$$

The case $\lambda = 0$ corresponds to the isothermal wall condition.

The work of Ergun [20] on fluid flow through packed columns is summarized as follows:

I. The laws of fluid flow through granular beds have several aspects of practical consequences. The total energy loss in fixed beds can be treated as the sum of viscous and kinetic energy losses.
II. The viscous energy losses per unit length are expressed by the term $\mu\frac{\partial u}{\partial y}$ and the kinetic energy losses by the term $2\rho K^* u\frac{\partial u}{\partial y}$. For $n = 1$, equation (4) reduces to

$$\mu\frac{\partial u}{\partial y}+2\rho K^* u\frac{\partial u}{\partial y}=K_p\rho g\beta\frac{\partial T}{\partial y}$$

The total loss has been balanced by the term on the right hand side of the above equation. The buoyancy induced flow of non-Newtonian fluid model for some representative exponents λ of the power-law wall temperature variation and viscosity index n has been considered following [13].

3. Solution of the problem

In order to solve the governing equations, we define the stream function $\psi(x,y)$such that

$$u=\frac{\partial\psi}{\partial y}, v=-\frac{\partial\psi}{\partial x} \tag{7}$$

The equation of continuity (3) is identically satisfied with this choice of u and v.

The equations (4) and (5) and the boundary conditions (6) become non-dimensional by introducing the following dimensionless variables and parameters.

$$\eta=\frac{y}{x}Pe_x^{1/2}\xi^{-1},\ \xi=\left[1+\sqrt{\frac{Ra_x}{Pe_x}}\right]^{-1},\\ f(\xi,\eta)=\frac{\psi(x,y)}{\alpha Pe_x^{1/2}}\xi,\ \theta(\xi,\eta)=\frac{T-T_\infty}{T_w-T_\infty},\ S=\frac{Qx^2\xi^2}{\alpha Pe_x} \tag{8}$$

where $Pe_x = \frac{U_\infty x}{\alpha}$ and $Ra_x = \frac{x}{\alpha}\left[\frac{\rho K_p g\beta}{\mu}(T_w - T_\infty)\right]^{\frac{1}{n}}$

Using equations (7) and (8) in equations (4) and (5), we have

$$n(f')^{n-1}f''+\frac{ErPe_d{}^n}{\xi^{2n}}2n(f')^{2n-1}f''=(1-\xi)^{2n}\theta' \tag{9}$$

$$\theta''+\frac{1}{2}\left[1+\frac{\lambda}{n}(1-\xi)\right]f\theta'-(\lambda f'-S)\theta\\ =\frac{\lambda}{2n}\xi(1-\xi)\left[\theta'\frac{\partial f}{\partial\xi}-f'\frac{\partial\theta}{\partial\xi}\right] \tag{10}$$

The corresponding boundary conditions are

$$\left.\begin{array}{l}\eta = 0 : f = 0,\ \theta = 1\\ \eta\to\infty : f'\to\xi^2,\ \theta\to 0\end{array}\right\} \tag{11}$$

where the primes denote differentiation with respect toη.

Here $Pe_d = \frac{U_\infty d}{\alpha}$ is the Peclet number based on the pore diameter, $Er = \frac{K^*}{\nu}\left[\frac{\alpha}{d}\right]^n$ is the Ergun number based on pore diameter, $Ra_d = \frac{d}{\alpha}\left[\frac{\rho K_p g\beta}{\mu}Ad^\lambda\right]^{1/n}$ is the Rayleigh number based on the pore diameter and $Ra_x = Ra_d\left(\frac{x}{d}\right)^{1+\frac{\lambda}{n}}$.

The computation has been carried out on the following basis:

From the definition, Pe_d is directly proportional to the pore diameter. In the present case($\lambda = 0$), Ra_x is inversely proportional to the pore diameter and when $x = d$, $Ra_x = Ra_d$. The scales of pore diameter and velocity are less than unity since $0<\xi<1$.

The non-Darcian velocity components u and v are

$$u=\frac{U_\infty}{\xi^2}f'(\xi,\eta) \tag{12}$$

$$v=-\frac{1}{2}\frac{\alpha}{x}Pe_x{}^{1/2}\frac{1}{\xi}\left[f+\frac{\lambda}{n}\left(1-\xi\right)f-\frac{\lambda}{n}\xi\left(1-\xi\right)\frac{\partial f}{\partial\xi}\\ +\eta\left(\left(-1+\frac{\lambda}{n}\left(1-\xi\right)\right)f'\right]\right. \tag{13}$$

For isothermal wall condition ($\lambda = 0$), equations (9) and (10) reduce to

$$n(f')^{n-1}f''+\frac{ErPe_d{}^n}{\xi^{2n}}2n(f')^{2n-1}f''=(1-\xi)^{2n}\theta' \tag{14}$$

$$\theta''+\frac{1}{2}f\theta'+S\theta=0 \tag{15}$$

with the boundary conditions

$$\left.\begin{array}{l}\eta=0 : f=0, \theta=1\\ \eta\to\infty : f'\to\xi^2, \theta\to 0\end{array}\right\}. \tag{16}$$

The heat transfer in terms of local Nusselt number $Nu_x$is given by

$$Nu_x Pe_x^{-1/2} = -\frac{1}{\xi}\theta'(\xi,0) \tag{17}$$

The skin friction coefficient C_f is given by

$$C_f = \frac{1}{2}\xi^3 C_f^*(Pr)^{-1}(Pe_x)^{1/2} = f''(\xi,0) \tag{18}$$

where $C_f{}^* = \frac{2\tau_w}{\rho U_\infty{}^2} = \frac{2Pr}{\xi^3\sqrt{Pe_x}}f''(\xi,0)$, $\tau_w = \mu\frac{\partial u}{\partial y}\Big]_{y=0}$ and $Pr = \mu/\rho\alpha$ (Prandtl number).

4. Results and discussion

The problem of non-Darcy mixed convective flow of non-Newtonian fluids on a vertical surface embedded in a porous medium with volumetric temperature dependent heat source has been considered. The present discussion is pertaining to the isothermal wall condition. Now, the resulting equations (14) and (15) along with the boundary conditions (16) are solved using fourth order Runge-Kutta method with a self corrective procedure (shooting technique). Most importantly, the method of solution under parametric and co-ordinate transformation play a vital role in paving the way to the solution.

Fig. 2(a) and (b) show the variation of velocity and temperature distributions across the flow domain for different values of Ergun number (Er) and heat source parameter (S) in case of pseudoplastic fluid ($n < 1$). The Ergun number (non-Darcy parameter) presents a complex structure depicting momentum diffusivity(ν), permeability of the medium (K_p), heat transport diffusivity (α) and pore diameter (d). Summarily, the effect of Ergun number decreases the velocity and increases the temperature across the flow domain slightly. The variation is asymptotic in nature to attain the free-stream condition that ensures the stability and laminarity of the flow. The decrease in velocity indicates that the momentum diffusivity (ν) and pore diameter (d) fail to encounter the effects of permeability of the medium and thermal diffusivity. Therefore, in such flow-model for higher values of Er, permeability and thermal diffusivity of the medium act proactively to reduce the velocity and enhance the temperature. Further, it is to note that an increase in heat source increases the velocity as well as the temperature.

Fig. 3(a) and (b) exhibit the effects of heat source and the Ergun number on velocity and temperature in the case of Newtonian fluid ($n = 1$). From Fig. 3(a) it is observed that with the increase in Er, there is a slight decrease in velocity distribution but no change is marked in the temperature distribution. This shows that the effect of non-Darcian parameter exhibited by Er is not so significant in a Newtonian fluid. Further, it is seen that presence of heat source increases both velocity and temperature of the fluid. Figures for dilatant fluid ($n > 1$) are omitted as those exhibit the same effect as that of Newtonian fluid.

Fig. 4(a) and (b) show the effect of Peclet number Pe_dwhich is an important parameter, a product of momentum diffusivity and thermal diffusivity, which regulates the two processes (momentum and thermal energy transport). It is seen that for small values of Pe_d, no significant change in asymptotic variation is marked on the velocity as well as temperature distribution. However, on careful observation, it is marked that an increase in Peclet number, decreases the velocity moderately since the Peclet number can be used in place of Reynolds number [21] due to the influence of the frictional and inertial forces on the flow field. For Newtonian as well as dilatant fluids, the coincidence of profiles is marked with that of pseudoplastic fluid (Figures omitted).

Fig. 5(a) and (b) show the effect of viscosity index parameter n on the velocity and temperature distribution. It is to note that increase in n, decreases significantly the velocity distribution, but the reverse effect is observed in case of temperature distribution which is in good agreement with [13]. The increasing or decreasing phenomena commensurate with the magnitude of the viscosity index, ignoring the linearity and non-linearity variation of viscosity which is a most interesting remark to note. This shows the fluidity or Rheology of the fluid model is at par with the viscosity index, from pseudoplastic ($n < 1$) to dilatant ($n > 1$) through Newtonian ($n = 1$) behavior of the fluid. Further, it is seen that an increase in n, increases the velocity and thermal boundary layer thickness.

Fig. 6(a) and (b) illustrates the effect of the mixed convection ξ parameter on the velocity and temperature profiles of pseudoplastic fluid. It is seen that as ξ increases, the velocity boundary layer increases, whereas the thermal boundary layer decreases. One interesting point is to note that buoyancy induced flow shows a significant variation near the vertical surface but remains constant afterwards.

Fig. 7 (a−c) exhibit the dimensionless wall shear stress, i.e. skin friction coefficient C_f for several values of S in cases of pseudoplastic, Newtonian and dilatant fluids respectively. From Fig. 7(a), it is seen that C_f increases abruptly for increase in heat source, but decreases slightly with the increase in heat sink. Thus, it

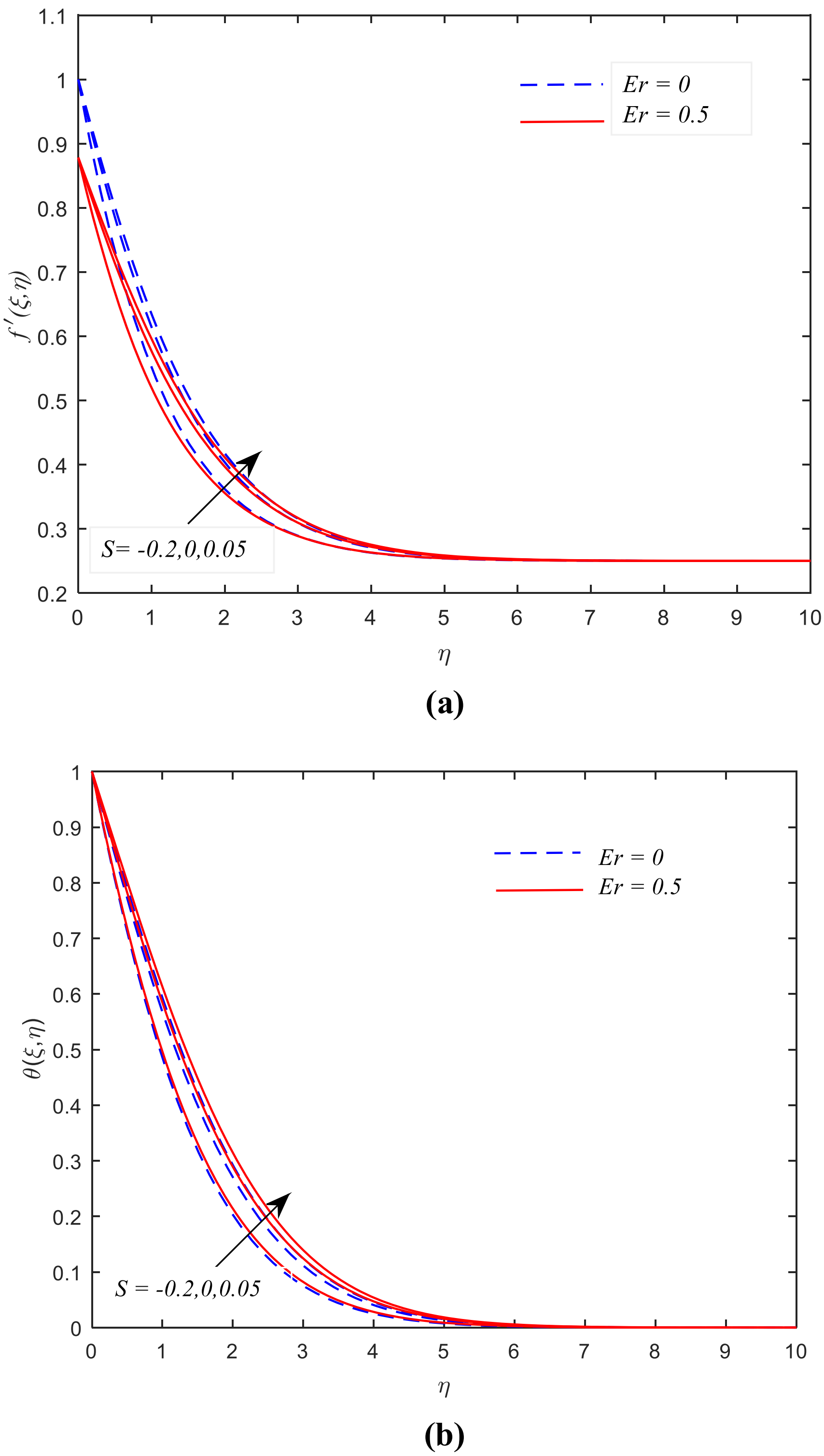
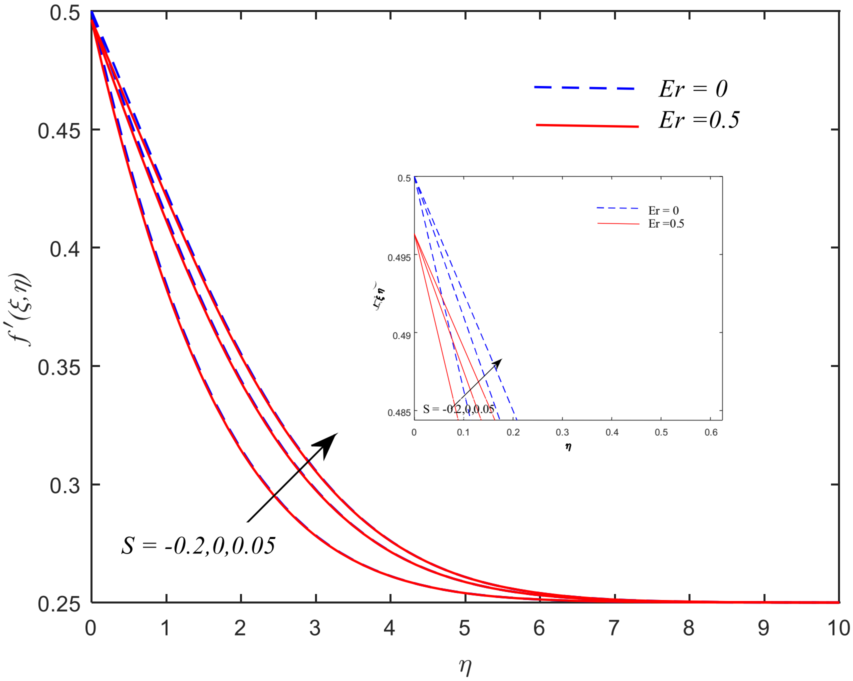
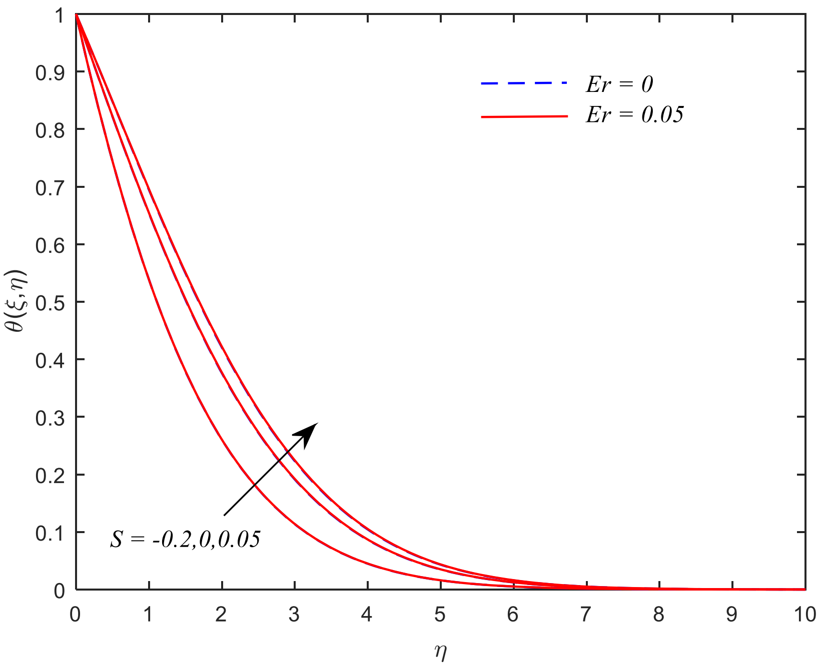


Fig. 2. (a, b). Velocity profile $f'(\xi,\eta)$and temperature profile $\theta(\xi,\eta)$for Er and S with fixed parameters: $Pe_d = 0.01$; $n = 0.5$ (pseudoplastic fluid); $\xi = 0.5$.



(a)



(b)

Fig. 3. (a, b). Velocity profile $f'(\xi,\eta)$and temperature profile $\theta(\xi,\eta)$for Er and S with fixed parameters: $Pe_d = 0.01$; $n = 1$(Newtonian fluid); $\xi = 0.5$.

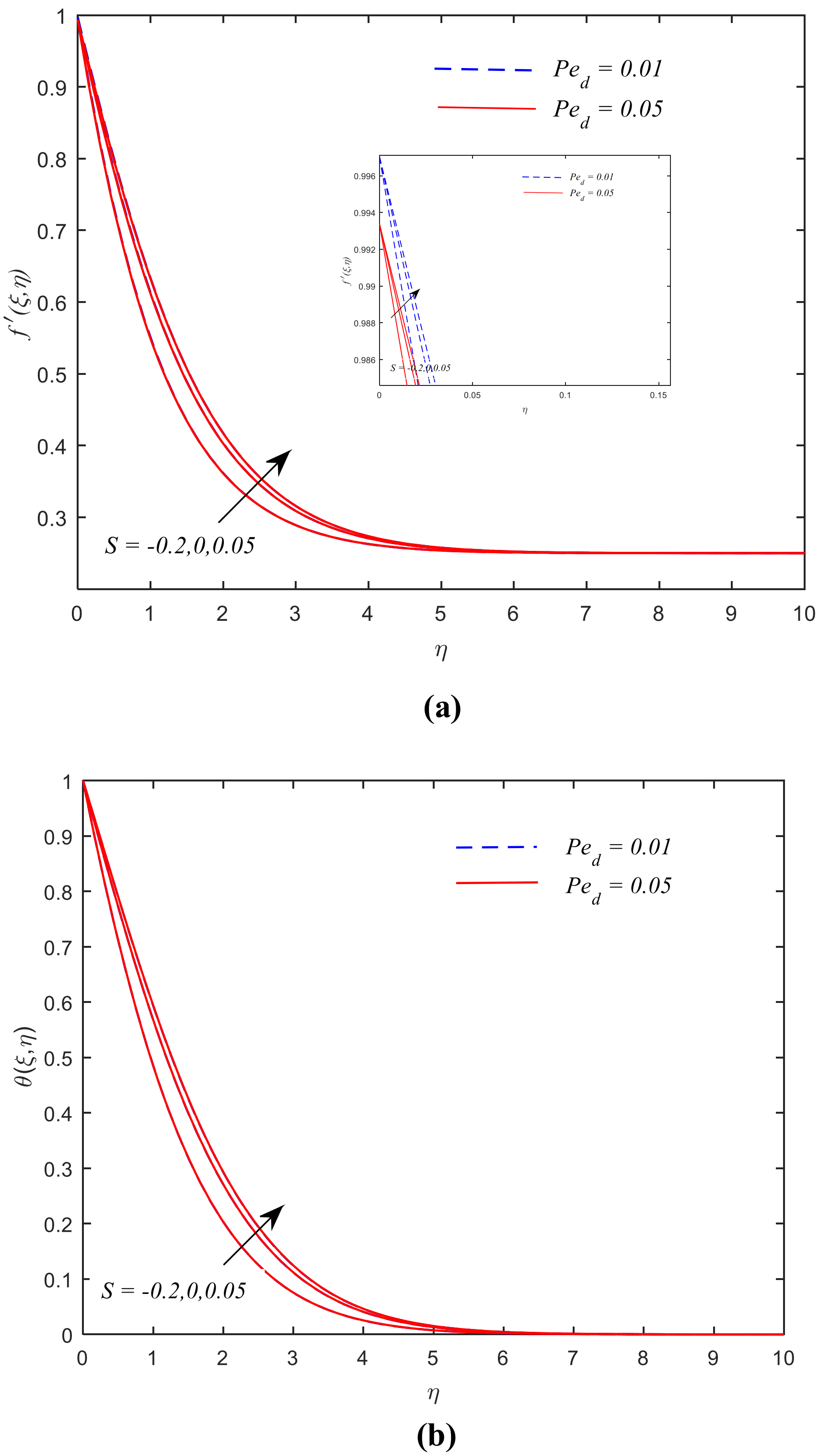


Fig. 4. (a, b). Velocity profile $f'(\xi,\eta)$ and temperature profile $\theta(\xi,\eta)$ for Pe_d and S with fixed parameters: $Er = 0.01$; $n = 0.5$ (pseudoplastic fluid); $\xi = 0.5$.

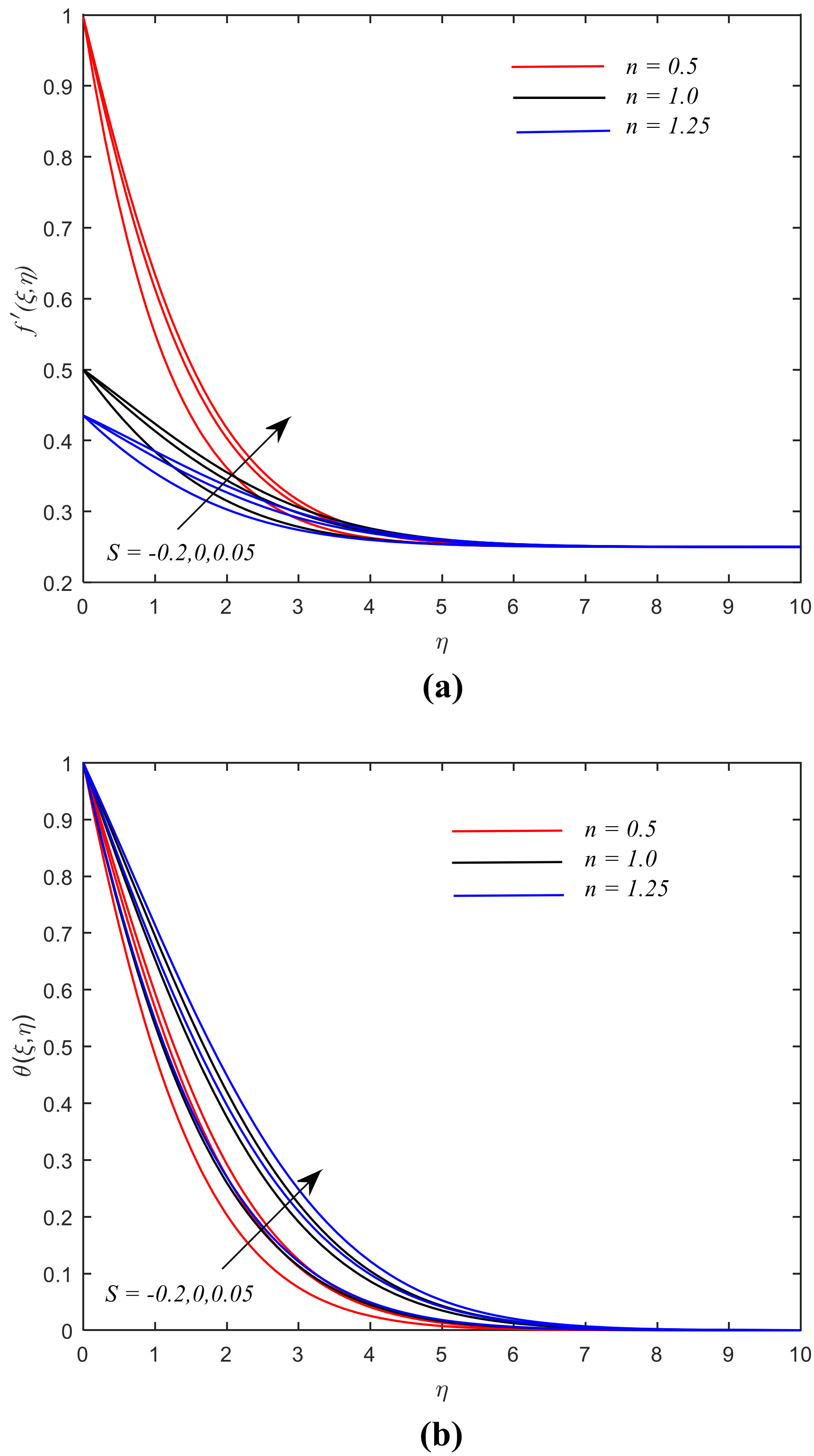


Fig. 5. (a, b). Velocity profile $f'(\xi,\eta)$and temperature profile $\theta(\xi,\eta)$for n and S with fixed parameters:$Er = 0.01; Pe_d = 0.01; \xi = 0.5$.

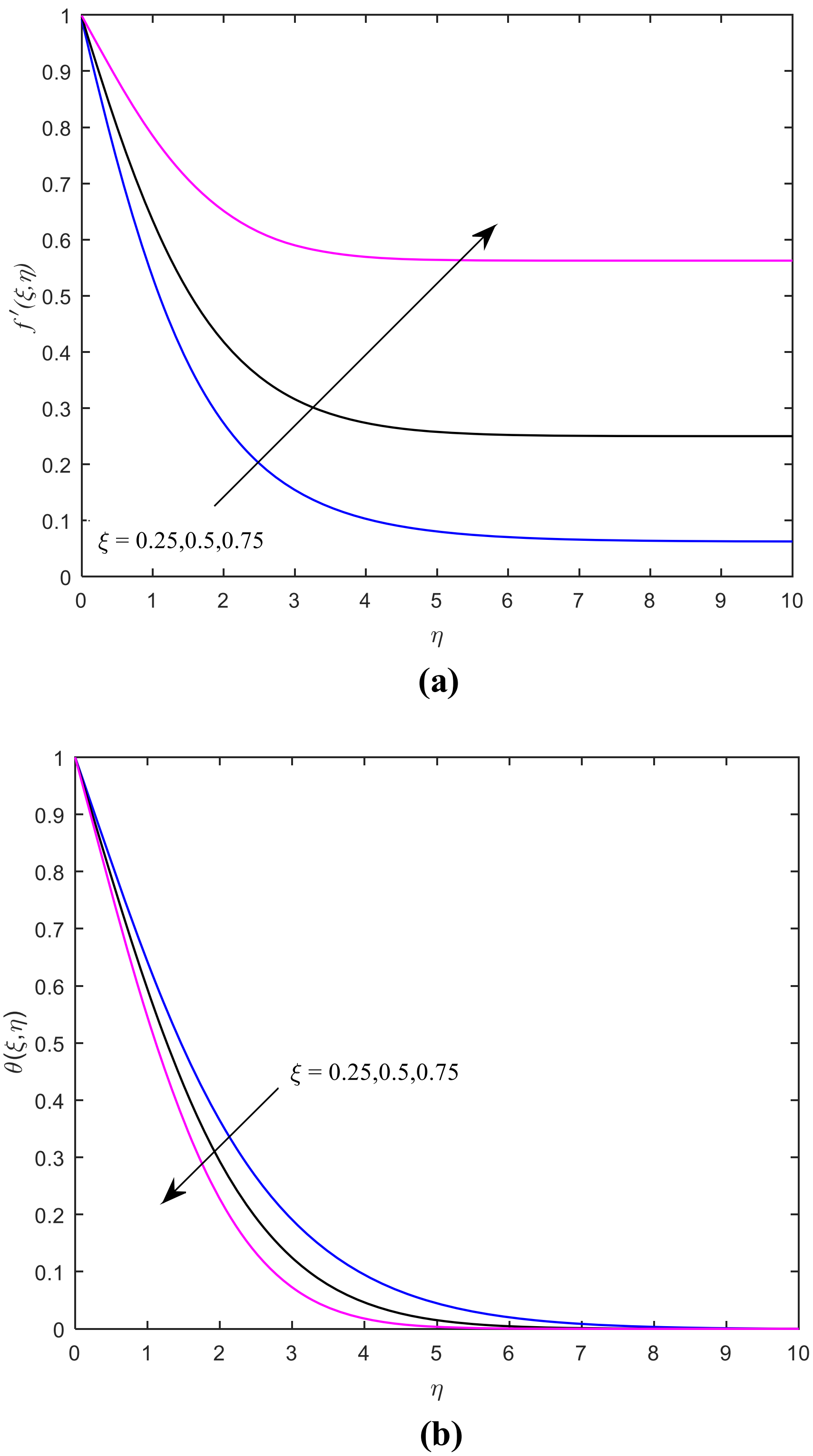
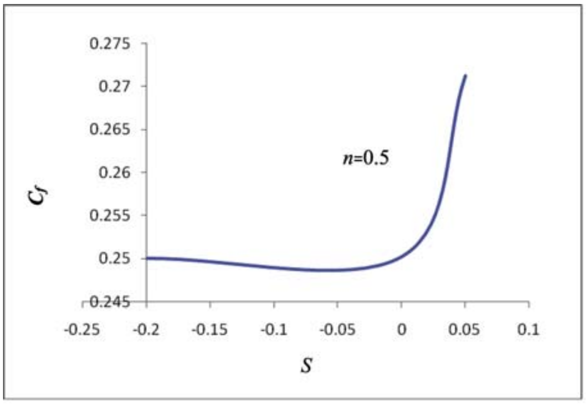
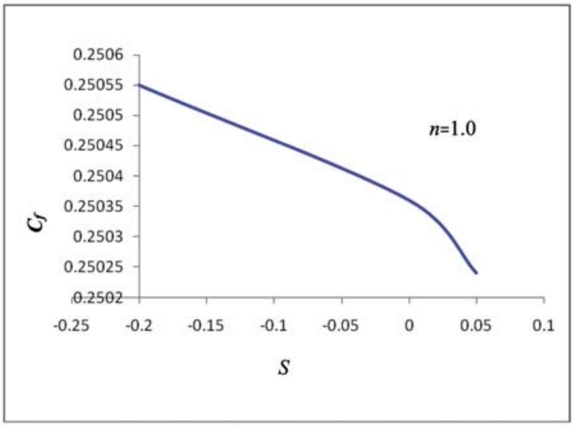


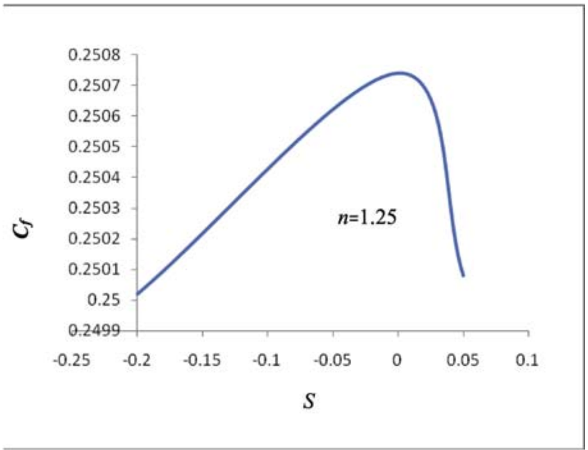
Fig. 6. (a, b). Velocity profile $f'(\xi,\ \eta)$and temperature profile $\theta(\xi,\ \eta)$for $\xi$with fixed parameters: $S = 0.05; Er = 0.01; Pe_d = 0.01; n = 0.5$ (pseudoplastic fluid).



(a)



(b)



(c)

Fig. 7. (a, b, c). Variation of skin friction C_f for S and n with fixed parameters: $Er = 0.01; Pe_d = 0.01; \xi = 0.5$.

Table 1
Rate of heat transfer at the wall $-\theta'(\xi,0)$

Er	Pe_d	n	S	$-\theta'(\xi,0)$
0.01	0.01	0.5	0.05	0.4152
0.02	—	—	—	0.4146
0.01	0.05	—	—	0.4145
—	0.01	1.0	—	0.2953
—	—	0.5	0.07	0.3945
—	—	1.25	0.05	0.2740

is concluded that for pseudoplastic fluid (viscosity decreases with an increase rate of shear), application of heat source is not suitable for yielding lower skin friction. In case of Newtonian fluid, steady decrease in skin friction is marked with the increase in heat sink ($S < 0$)/source ($S > 0$) parameter. For dilatant fluid (apparent viscosity increase with increase in shear rate), sharp rise and fall in C_f are marked with heat sink and source, respectively. Hence, the presence of the heat sink is not desirable in dilatant fluid flow to avoid the escalation in skin friction. To conclude, for the steady decrease in surface shear stress, the Newtonian fluid is suitable otherwise, the choice of fluid depends on the design requirement.

Table 1 depicts the Nusselt number, a bounding surface heat-transfer criteria, responsible for heating or cooling of the surface as per the design requirement for mixed convection flow. From the table it is observed that $-\theta'(\xi,0) > 0$ indicates heat flows from the plate to the fluid mass. Further, it is seen that an increase in Er, Pe_d, n and S decelerate the surface cooling from pseudoplasticity to dilatancy through Newtonian. Thus, it is concluded that enhancing Er (the Ergun number which depends upon the pore diameter and inertial coefficient), Pe_d (ratio of heat transfer by convection to conduction), S (strength of the volumetric heat source) and viscosity index parameter n, slows down the cooling of the bounding surface.

Table 2 shows the validity of the present result of the R−K method with that of the Finite Difference

Table 2
Comparison table.

ξ	n	S	Er	Pe_d	Present result (R−K method)	Kumari and Jayanthi [13] (Finite Difference method)
					$-\theta'(\xi,0)$	$-\theta'(\xi,0)$
0.25	0.5	0	0	0.01	0.4186	0.4171
0.5	—	—	—	—	0.4648	0.4632
0.75	—	—	—	—	0.5136	0.5131
0.25	1.0	—	—	—	0.3626	0.3615
0.5	—	—	—	—	0.3603	0.3596
0.75	—	—	—	—	0.4378	0.4352

method of [13]. It is found that the results are in good agreement.

5. Conclusion

Ergun number reduces the velocity boundary layer of pseudoplastic fluid, a desirable outcome, but enhances the thermal boundary layer, whereas, in case of Newtonian and dilatant fluid, the effect is not so significant. Increase in Peclet number (a product of Reynolds number and Prandtl number) decreases the velocity of all types of fluid moderately due to inertia and frictional forces. Viscosity index of the fluid affects the flow near the solid surface significantly. The mixed convective parameter gives rise to the thicker velocity boundary layer, whereas, opposite effect is observed in case of the thermal boundary layer. Increase in viscosity index decelerates the rate of cooling of the bounding surface. Significant reduction, smooth variation and hike in skin friction mainly depend upon pseudoplasticity, Newtonian or dilatancy of fluid in the presence of heat source.

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