Karbala International Journal of Modern Science

Volume 7 | Issue 1 Article 5

Third- and Fifth-Order Geometric Aberrations in Magnetic Exponential Lens Model for Object Magnetic Immersion

Aymen Amer Mr.
Al Nahrain Univrsity, aymenamer1993@gmail.com

Ahmad K. Ahmad Professor
Al_Nahrain University, ahmad.ahmad@ced.nahrainuniv.edu.iq

Recommended Citation
Amer, Aymen Mr. and Ahmad, Ahmad K. Professor (2021) "Third- and Fifth-Order Geometric Aberrations in Magnetic Exponential Lens Model for Object Magnetic Immersion," *Karbala International Journal of Modern Science*: Vol. 7 : Iss. 1 , Article 5.
Available at: https://doi.org/10.33640/2405-609X.2235

Third- and Fifth-Order Geometric Aberrations in Magnetic Exponential Lens Model for Object Magnetic Immersion

Abstract

We numerically employ the differential algebraic technique to calculate the third and fifth-order geometric aberrations coefficients, which are derived by using the map method, of the magnetic exponential lens model. These coefficients are calculated for object magnetic immersion (OMI). The magnetic exponential model is used as an example for the magnetic round lens to calculate the coefficients. The numerical electron optical results are perfectly in match with the analytically results with a very small relative error (10^{-10}- 10^{-12}). The DA advantage technique is a very compendious, effective, proper, and accurate for analysis of electron optical aberration. The COSYINFINITY 10 code is used in our calculations.

Keywords

Differential algebra, Map method, Magnetic round lens, high order Geometric aberrations, OMI effect, COSYINFINITY 10

Cover Page Footnote

The authors come forward with gratitude to Professor M. Berz and Professor Kyoko Makino for providing the license and the valuables note about the program COSY INFINITY 10 for this work. Our gratitude also to Professor Zhixiong Liu for the valuable advice he gave us during the work. The authors acknowledge the College of Science at Al-Nahrain University for support through the work.

1. Introduction

Through the growing applications of high-quality systems of focusing charged particle beams such as high-resolution electron microscopes, electron and ion lithography systems, and beam accelerators, the desire to design practical electromagnetic lenses with aberrations of higher orders than third-order is receiving international attention [1−3]. Therefore, it is required to study the higher-order aberration for such focusing systems. Differential algebra (DA) is a highly accurate and advanced tool for solving sets of nonlinear dynamical equations as in the high-order aberrations of systems of electron-optical lenses. The DA method is a very accurate method with no special requirements for the calculation, developed by many authors including Berz [4], Wang and Chen [5,6] and Liu [7]; it can be applied to problems in the electric and/or magnetic fields engineering design. In this work, we utilize the DA method and COSY INFINITY 10 program to calculate practical electron lenses with magnetic fields that include object magnetic immersion (OMI) corrections. The OMI correction method can be used to correct chromatic and spherical aberration coefficients for magnetic lenses when the object is located in the field of the magnetic lens. The Gaussian properties and the third and fifth orders aberrations are determined for an exponential magnetic electron lens whose magnetic field magnitudes are input into a discretized array structure for further analysis. Three DA variable types exist for analyzing the aberration of rotationally symmetric electron optical systems: the fixed coordinates DA description, the rotating coordinates DA description, and the hybrid coordinates DA description [7,8]. The rotational coordinate system is used to facilitate the analytical paraxial ray equation. In the magnetic lens case, if the object immersed in the field, the correction of aberration done using OMI effect [9]. In the case of higher-order aberration (Fifth and Seventh), the description of DA in rotational coordinates becomes very complicated. We derive the general equation of electron trajectory and we obtain the transfer map of the rotational coordinates by tracking the trajectory equation. Seman [10] was the first researcher who discussed the problems of object magnetic immersion OMI in magnetic lenses. The high order aberrations (third and fifth) for the magnetic exponential model are carried out using the DA technique. In this work, the DA technique using COSY INFINITY 10 [11,12] is used for such calculations for the first time. The DA results were crosschecked using the high order (third and fifth) aberration integrals evaluated by Wolfram Mathematica 9 program [13]. The results proved to be very compendious, effective, and accurate. The expression of the general equation of electron trajectory in rotating and fixed coordinates are given in many references [14−19]. It performs a significant part in clarifying the differential algebraic method in rotational coordinates.

2. DA description types of high order geometrical aberrations

Tracking the equation of the general trajectory of the electrons is used in fixed coordinates to obtain the higher orders (third and fifth) geometric aberrations using the deferential algebra integrator limit starting at z_o (plane of object) to z_i (plane of image) in fixed coordinate [18];

$$\begin{aligned}
\Delta X_{3i} &= \sum_{k.l.m.n=0.1.2.3}^{k+l+m+n=3} M_f(1.klmn) X_o^k X'^l_o Y_o^m Y'^n_o, \\
\Delta Y_{3i} &= \sum_{k.l.m.n=0.1.2.3}^{k+l+m+n=3} M_f(3.klmn) X_o^k X'^l_o Y_o^m Y'^n_o, \\
\Delta X_{5i} &= \sum_{k.l.m.n=0.1.2.3.4.5}^{k+l+m+n=5} M_f(1.klmn) X_o^k X'^l_o Y_o^m Y'^n_o, \\
\Delta Y_{5i} &= \sum_{k.l.m.n=0.1.2.3.4.5}^{k+l+m+n=5} M_f(3.klmn) X_o^k X'^l_o Y_o^m Y'^n_o
\end{aligned} \tag{1}$$

where M_f (i, klmn) is the transfer map elements in fixed coordinates, the number of orders depending on the sum of k, l, m, and n. While tracking the expression of rotating transform coordinates the DA characterization of rotational coordinates are calculated [16,18] as in the following form;

$$\begin{aligned}
\Delta x_{3i} &= \sum_{k.l.m.n=0.1.2.3}^{k+l+m+n=3} M_r(1.klmn) x_o^k x'^l_o y_o^m y'^n_o, \\
\Delta y_{3i} &= \sum_{k.l.m.n=0.1.2.3}^{k+l+m+n=3} M_r(3.klmn) x_o^k x'^l_o y_o^m y'^n_o, \\
\Delta x_{5i} &= \sum_{k.l.m.n=0.1...5}^{k+l+m+n=5} M_r(1.klmn) x_o^k x'^l_o y_o^m y'^n_o, \\
\Delta y_{5i} &= \sum_{k.l.m.n=0.1....5}^{k+l+m+n=5} M_r(1.klmn) x_o^k x'^l_o y_o^m y'^n_o
\end{aligned} \tag{2}$$

https://doi.org/10.33640/2405-609X.2235

where M_r (i, klmn) is the transfer map elements in the rotating coordinates. The third-order geometrical aberrations have an expansion, in the vector form, of combined electromagnetic lenses

$$\Delta\vec{r}_3=B\left[\vec{r}_o{}'\cdot\vec{r}_o{}'\right]\vec{r}_o{}'+F\left[\left(\vec{r}_o{}'\cdot\vec{r}_o{}'\right)\vec{r}_o+2\left(\vec{r}_o{}'\cdot\vec{r}_o\right)\vec{r}_o\right]+2C\left[\vec{r}_o{}'\cdot\vec{r}_o\right]\vec{r}_o+D\left[\vec{r}_o\cdot\vec{r}_o\right]\vec{r}_o{}'$$
$$+E\left[\vec{r}_o\cdot\vec{r}_o\right]\vec{r}_o+f\left[3\left(\vec{r}_o{}'\cdot\vec{r}_o{}'\right)\vec{r}'^{*}_o-2\left(\vec{r}_o{}'\cdot\vec{r}_o\right)\vec{r}'^{*}_o\right]+c\left[2\left(\vec{r}_o{}'\cdot\vec{r}_o\right)\vec{r}'^{*}_o-\left(\vec{r}_o.\vec{r}_o\right)\vec{r}'^{*}_o\right] \quad (3)$$
$$+e\left[\left(\vec{r}_o.\vec{r}_o\right)\vec{r}^{*}_o\right]$$

$$\vec{r}_o=(x_o.y_o),\quad \vec{r}_o{}'=(x_o{}'.y_o{}'),\quad \vec{r}^{*}_o=(-y_o.x_o),\quad \vec{r}'^{*}_o=(-y_o{}'.x_o{}')$$

In addition, the expansion, for combined electromagnetic lenses, of the fifth order geometrical aberrations in a vector form is;

$$\Delta\vec{r}_{5i}=\left(\vec{r}_o{}'\bullet\vec{r}_o{}'\right)^2\left(A_5\vec{r}_o{}'+a_5\vec{r}'^{*}_o\right)+\left(\vec{r}_o{}'\bullet\vec{r}_o{}'\right)^2\left(B_{51}\vec{r}_o+b_{51}\vec{r}^{*}_o\right)$$
$$+(\vec{r}_o{}'\bullet\vec{r}_o{}')\left(\vec{r}_o{}'\bullet\vec{r}_o\right)\left(B_{52}\vec{r}_o{}'+b_{52}\vec{r}'^{*}_o\right)+\left(\vec{r}_o{}'\bullet\vec{r}_o{}'\right)\left(\vec{r}_o{}'\bullet\vec{r}_o\right)\left(C_{51}\vec{r}_o+c_{51}\vec{r}^{*}_o\right)$$
$$+(\vec{r}_o{}'\bullet\vec{r}_o{}')\left(\vec{r}_o\bullet\vec{r}_o\right)\left(C_{52}\vec{r}_o{}'+c_{52}\vec{r}'^{*}_o\right)+\left(\vec{r}_o{}'\bullet\vec{r}_o\right)^2\left(C_{53}\vec{r}'_o+c_{53}\vec{r}'^{*}_o\right)$$
$$+(\vec{r}_o{}'\bullet\vec{r}_o{}')\left(\vec{r}_o\bullet\vec{r}_o\right)\left(D_{51}\vec{r}_o+d_{51}\vec{r}^{*}_o\right)+\left(\vec{r}_o{}'\bullet\vec{r}_o\right)^2\left(D_{52}\vec{r}_o+d_{52}\vec{r}^{*}_o\right)+ \quad (4)$$
$$\left(\vec{r}_o{}'\bullet\vec{r}_o\right)\left(\vec{r}_o\bullet\vec{r}_o\right)\left(D_{53}\vec{r}_o{}'+d_{53}\vec{r}'^{*}_o\right)+\left(\vec{r}_o{}'\bullet\vec{r}_o\right)\left(\vec{r}_o\bullet\vec{r}_o\right)\left(E_{51}\vec{r}_o+e_{51}\vec{r}^{*}_o\right)$$
$$+\left(\vec{r}_o\bullet\vec{r}_o\right)^2\left(E_{52}\vec{r}_o{}'+e_{52}\vec{r}'^{*}_o\right)+\left(\vec{r}_o\bullet\vec{r}_o\right)^2\left(F_5\vec{r}_o+f_5\vec{r}^{*}_o\right),$$
$$\vec{r}_o=(x_o.y_o),\vec{r}_o{}'=(x_o{}'.y_o{}'),\vec{r}^{*}_o=(-y_o.x_o),\quad \vec{r}'^{*}_o=(-y_o{}'.x_o{}'),$$

In comparison equation (2) with equations (3) and (4), the connection between the elements of the map in Eq. (2) and the coefficients of aberration in equations (3) and (4) are obtained. The Glaser's notation of the third order coefficients of the geometrical aberration are [14]; $B=M_r(1.0300), F=M_r(1.1002),\ C=\frac{M_r(1.1011)}{2}, D=M_r(1.0120),\ E=M_r(1.3000), f=M_r(3.1002)/3, c=M_r(3.1011)/2, e=M_r(3.3000)$ and the fifth order coefficients are, $A_5=M_r(1.0500), B_{51}=M_r(1.1004),\ B_{52}=M_r(1.0311),\ C_{51}=M_r(1.1013),\ C_{52}=M_r(1.0320).$ $C_{53}=M_r(1.0122)-C_{52},\ D_{51}=M_r(1.3002),\ D_{52}=M_r(1.1022)-D_{51},\ D_{53}=M_r(1.0131),\ E_{51}=M_r(1.3011),\ E_{52}=M_r(1.0140),\ F_5=M_r(1.5000),\ a_5=M_r(3.0500),\ b_{51}=M_r(3.1004),\ b_{52}=M_r(3.0311),\ c_{51}=M_r(3.1013),\ c_{52}=M_r(3.0320),$ $c_{53}=M_r(3.0122)-c_{52},\ d_{51}=M_r(3.3002),\ d_{52}=M_r(3.1022)-d_{51},\ d_{53}=M_r(3.0131),\ e_{51}=M_r(3.3011),\ e_{52}=M_r(3.0140),\ f_5=M_r(3.5000).$

The hybrid coordinates DA description have the form [3],

$$\begin{aligned}\Delta x_{3mi} &= \sum_{k.l.m.n=0.1.2.3}^{k+l+m+n=3} M_h(1.klmn) X_o^k X'^l_o Y_o^m Y'^n_o,\\ \Delta y_{3mi} &= \sum_{k.l.m.n=0.1.2.3}^{k+l+m+n=3} M_h(3.klmn) X_o^k X'^l_o Y_o^m Y'^n_o,\\ \Delta x_{5mi} &= \sum_{k.l.m.n=0.1.2.3.4.5}^{k+l+m+n=5} M_h(1.klmn) X_o^k X'^l_o Y_o^m Y'^n_o,\\ \Delta x_{5mi} &= \sum_{k.l.m.n=0.1.2.3.4.5}^{k+l+m+n=5} M_h(3.klmn) X_o^k X'^l_o Y_o^m Y'^n_o\end{aligned} \tag{5}$$

where the suffix "m" indicates using the effect of OMI, the aberrations were corrected. The hybrid coordinates transfer map elements are represented by $M_h(1.klmn)$ and $M_h(3.klmn)$, where they could be derived from $M_f(1.klmn)$ and $M_f(3.klmn)$. The relation between the geometrical aberration coefficients and the hybrid coordinates map elements together with the containment of the effect of OMI is calculated. The aberration coefficients of the third order isotropic are:

$$\begin{aligned}&B_m = M_h(1.0300) = M_f(3.0300)\sin\theta_i + M_f(1.0300)\cos\theta_i\\ &F_m = M_h(1.1002) = M_f(3.1002)\sin\theta_i + M_f(1.1002)\cos\theta_i\\ &C_m = M_h(1.1011) = \left[M_f(3.1011) sin\theta_i + M_f(1.1011)\cos\theta_i\right]/2\\ &D_m = M_h(1.0120) = M_f(3.0120)\sin\theta_i + M_f(1.0120)\cos\theta_i\\ &E_m = M_h(1.3000) = M_f(3.3000)\sin\theta_i + M_f(1.3000)\cos\theta_i\end{aligned} \tag{6}$$

The aberration coefficients of the third order anisotropic are:

$$\begin{aligned}&f_m = M_h(3.1002) = \left[-M_f(1.1002)\sin\theta_i + M_f(3.1002)\cos\theta_i\right]/3\\ &c_m = M_h(3.1011) = \left[-M_f(1.1011)\sin\theta_i + M_f(3.1011)\cos\theta_i\right]/2\\ &e_m = M_h(3.3000) = \left[-M_f(1.3000)\sin\theta_i + M_f(3.3000)\cos\theta_i\right]\end{aligned} \tag{7}$$

The aberration coefficients of the fifth order isotropic are:

$$\begin{aligned}&A_{5m} = M_h(1,0500) = M_f(3,0500) sin\theta_i + M_f(1,0500) cos\theta_i\\ &B_{51m} = M_h(1,1004) = M_f(3,1004) sin\theta_i + M_f(1,1004) cos\theta_i\\ &B_{52m} = M_h(1,0311) = M_f(3,0311) sin\theta_i + M_f(1,0311) cos\theta_i\\ &C_{51m} = M_h(1,1013) = M_f(3,1013) sin\theta_i + M_f(1,1013) cos\theta_i\\ &C_{52m} = M_h(1,0320) = M_f(3,0320) sin\theta_i + M_f(1,0320) cos\theta_i\end{aligned} \tag{8}$$

$$\begin{aligned}&C_{53m} = M_h(1,0122) - C_{52m} = M_f(3,0122) sin\theta_i + M_f(1,0122) cos\theta_i - C_{52m}\\ &D_{51m} = M_h(1,3002) = M_f(3,3002) sin\theta_i + M_f(1,3002) cos\theta_i\\ &D_{52m} = M_h(1,1022) - D_{51m} = M_f(3,1022) sin\theta_i + M_f(1,1022) cos\theta_i - D_{51m}\\ &D_{53m} = M_h(1,0131) = M_f(3,0131) sin\theta_i + M_f(1,0131) cos\theta_i\\ &E_{51m} = M_h(1,3011) = M_f(3,3011) sin\theta_i + M_f(1,3011) cos\theta_i\\ &E_{52m} = M_h(1,0140) = M_f(3,0140) sin\theta_i + M_f(1,0140) cos\theta_i\\ &F_{5m} = M_h(1,5000) = M_f(3,5000) sin\theta_i + M_f(1,5000) cos\theta_i\end{aligned} \tag{9}$$

The aberration coefficients of the fifth order anisotropic are:

$$\begin{aligned}&a_{5m} = M_h(3,0500) = -M_f(1,0500) sin\theta_i + M_f(3,0500) cos\theta_i\\ &b_{52m} = M_h(3,0311) = -M_f(1,0311) sin\theta_i + M_f(3,0311) cos\theta_i\\ &c_{51m} = M_h(3,1013) = -M_f(1,1013) sin\theta_i + M_f(3,1013) cos\theta_i\\ &c_{52m} = M_h(3,0320) = -M_f(1,0320) sin\theta_i + M_f(3,0320) cos\theta_i\end{aligned} \tag{10}$$

$$\begin{aligned}&c_{53m} = M_h(3,0122) - c_{52m} = -M_f(1,0122) sin\theta_i + M_f(3,0122) cos\theta_i - c_{52m}\\ &d_{51m} = M_h(3.3002) = -M_f(1.3002)\sin\vartheta_i + M_f(3.0300)\cos\vartheta_i\\ &d_{52m} = M_h(3.1022) - d_{51m} = -M_f(1.1022)\sin\vartheta_i + M_f(3.1022)\cos\vartheta_i - d_{51m}\\ &d_{53m} = M_h(3.0131) = -M_f(1.0131)\sin\vartheta_i + M_f(3.0131)\cos\vartheta_i\\ &e_{51m} = M_h(3.3011) = -M_f(1.3011)\sin\vartheta_i + M_f(3.3011)\cos\vartheta_i\\ &e_{52m} = M_h(3.0140) = -M_f(1.0140)\sin\vartheta_i + M_f(3.0140)\cos\vartheta_i\\ &f_{5m} = M_h(3.5000) = -M_f(1.5000)\sin\vartheta_i + M_f(3.5000)\cos\vartheta_i\end{aligned} \tag{11}$$

3. High order aberrations of the magnetic lens

The concept of higher-order aberration coefficients derivation is the same as that in Refs. [18,19]. According to equation (3), there are three anisotropic (lowercase letters) and five isotropic (uppercase letters) coefficients of the third-order aberration: the symbol B refers to the spherical aberration, the symbols F and f refers to coma, the symbols C and c refers to astigmatism, the symbol D refers to the field curvature, and the symbols E and e refers to the distortion. According to Equation (4), there are twelve isotropic and twelve anisotropic fifth-order coefficients, where the uppercase symbols are for isotropic and the lowercase symbols are for anisotropic coefficients. For spherical aberration we use the symbols a_5 and A_5, the symbols b_{51}, $b_{52,}$ and B_{51}, B_{52} for coma, the symbols $c_{51}.c_{52}.c_{53}$ and C_{51}, C_{52}, C_{53} for the peanut aberration, the symbols $d_{51}.d_{52}.d_{53} and D_{51}, D_{52}, D_{53}$ for the elliptical coma, the symbols $e_{51} and E_{51}$ for astigmatism, the symbols $e_{52} and\ E_{52}$ for field curvature, and finally the symbols $f_5 and F_5$ refer to the distortion [20−22]. The high order coefficients formulas for OMI correction are given as follows;

$$\mathrm{B_m} = \mathrm{B},\ \mathrm{F_m} = \mathrm{F}, \qquad \mathrm{C_m} = \mathrm{C} + 2\theta_o{}' f + \theta_o'^2 \mathrm{B},$$
$$D_m = D - 6\theta_o{}' f + 3\theta_o'^2 B, \qquad E_m = E - 2\theta_o{}' c + \theta_o'^2 F.$$
$$f_m = f - \theta_o{}' B,\ c_m = c - \theta_o{}' F. \ e_m = e - \theta_o{}' D + 3\theta_o'^2 f - \theta_o'^3 B \tag{12}$$

$$\begin{aligned}
&A_{5m} = A_5, \quad B_{51m} = B_{51} + 5\theta_o{}' a_5,\\
&B_{52m} = B_{52} - 4\theta_o{}' a_5,\\
&C_{51m} = C_{51} + \theta_o{}'(4b_{51} + 3b_{52}) - 4\theta_o'^2 A_5,\\
&C_{52m} = C_{52} - 4\theta_o{}' b_{51} + 10\theta_o'^2 A_5,\\
&C_{53m} = C_{53} - 2\theta_o{}' b_{52} - 4\theta_o'^2 A_5,\\
&D_{51m} = D_{51} + 3\theta_o{}' c_{52} + 6\theta_o'^2 B_{51} + 10\theta_o'^3 a_5,\\
&D_{52m} = D_{52} + \theta_o{}'(2c_{51} + c_{53}) - 2\theta_o'^2(2B_{51} + B_{52}) - 4\theta_o'^3 a_5,\\
&D_{53m} = D_{53} - 2\theta_o{}'(c_{51} + c_{53}) - 3\theta_o'^2 B_{52} - 4\theta_o'^3 a_5,\\
&E_{51m} = E_{51} + \theta_o{}'(2d_{51} + d_{53}) + \theta_o'^2(C_{51} - 2C_{52}) + \theta_o'^3(4b_{51} + b_{52}) - 4\theta_o'^4 A_5,\\
&E_{52m} = E_{52} - 2\theta_o{}' d_{51} + 3\theta_o'^2 C_{52} - 4\theta_o'^3 b_{51} + 5\theta_o'^4 A_5,\\
&F_{5m} = F_5 + \theta_o{}' e_{52} + \theta_o'^2 D_{51} + \theta_o'^3 c_{52} + \theta_o'^4 B_{51} + \theta_o'^5 a_5;
\end{aligned} \tag{13}$$

$$\begin{aligned}
&a_{5m} = a_5, \quad b_{51m} = b_{51} - 5\theta_o{}' A_5, b_{52m} = b_{52} + 4\theta_o{}' A_5,\\
&c_{51m} = c_{51} - \theta_o{}'(4B_{51} + 3B_{52}) - 4\theta_o'^2 a_5,\\
&c_{52m} = c_{52} + 4\theta_o{}' B_{51} + 10\theta_o'^2 a_5,\\
&c_{53m} = c_{53} + 2\theta_o{}' B_{52} - 4\theta_o'^2 a_5,\\
&d_{51m} = d_{51} - 3\theta_o{}' C_{52} + 6\theta_o'^2 b_{51} - 10\theta_o'^3 A_5,\\
&d_{52m} = d_{52} - \theta_o{}'(2C_{51} + C_{53}) - 2\theta_o'^2(2b_{51} + b_{52}) + 4\theta_o'^3 A_5,\\
&d_{53m} = d_{53} + 2\theta_o{}'(C_{51} + C_{52}) + 3\theta_o'^2 b_{52} + 4\theta_o'^3 A_5,\\
&e_{51m} = e_{51} - \theta_o{}'(2D_{51} + D_{53}) + \theta_o'^2(c_{51} - 2c_{52}) + \theta_o'^3(4B_{51} + B_{52}) - 4\theta_o'^4 a_5,\\
&e_{52m} = e_{52} + 2\theta_o{}' D_{51} + 3\theta_o'^2 c_{52} + 4\theta_o'^3 B_{51} + 5\theta_o'^4 a_5,\\
&f_{5m} = f_5 - \theta_o{}' E_{52} + \theta_o'^2 d_{51} - \theta_o'^3 C_{52} + \theta_o'^4 b_{51} - \theta_o'^5 A_5.
\end{aligned} \tag{14}$$

where $\theta_o{}' = \frac{\eta}{2}\frac{B(z_o)}{\sqrt{V(z_o)}}$, $\eta = \sqrt{\frac{e}{2m}}$, $B(\mathrm{z_o})$ and $V(\mathrm{z_o})$ are the magnetic induction and potential function respectively at the object plane $\mathrm{z_o}$.

4. Computational illustration

The former derivations were a concise description of the basics of the high order geometric aberrations calculation for electron lenses with magnetic fields that include OMI corrections. More information about the DA method can be found in Berz (1999) [22]. The formulation of the DA method is independent of the aberration order, thus, to understand the form of the Hamiltonian an analytical description of the field must be defined. Thus, we must know the exact form of the axial potentials and their derivatives, to determine any order of aberrations. Therefore, the inaccuracy of the axial potential higher-order derivatives is a limit for the use of the DA method in real optical systems, i.e. the analytical field model (or the axil potential) must have a trajectory equation that can be solved analytically and its higher order aberrations can also be expressed by the analytical expression of the field model to have an accurate solution [23,24].

In the present work, the exponential model of the magnetic round lens is one of the well-known models that can make the paraxial ray equation soluble and useful, especially for 'single-pole' lenses [25], whose magnetic induction distributions have the following form

$$B(z) = B_0 e^{-\left[\frac{z}{d}\right]^2}. \tag{15}$$

where $B_o = 0.01$ T is the maximum magnetic flux and d is the half-width of the magnetic flux distribution in mm. The axial magnetic flux distributions are B(z) shown in Fig. 1, along z (optical axis) from $z_o = -0.3$ mm (object plane) to $z_i = 0.3$ mm (image plane) at different values of d ($d = 0.1, 0.09, 0.08, 0.07, 0.06, 0.05$ mm). Two packages are used: the COSY INFINITY10 [11,12,26] and Wolfram Mathematica version 9 [13]. With the aid of the 8th order Runge-Kutta integrator [27], we calculate the transfer map produced by tracking the trajectory equations in rotating and fixed coordinates. All the high order aberration coefficients (third and fifth) are calculated

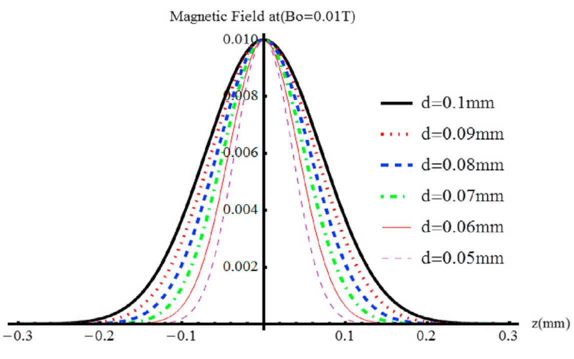

Fig. 1. The axial magnetic fields B(z) as a function of different values of (d) along the optical axis (z) with $B_o = 0.01$ T.

using two packages (without and with the OMI participation) for the magnetic exponential model. The spherical aberration disc Δr (disc of least confusion) is also calculated using the following equation [28,29]:

$$\Delta r = MB\alpha_o^3. \tag{16}$$

where M is the magnification, B (in some reference denoted by C_s) is the third order spherical aberration coefficient and α_o is the half angle in radian and we take a range of $(\alpha_o)^3$ from zero to 0.2 radian.

5. Discussion and conclusion

The third- and fifth-order isotropic and anisotropic geometric aberration with and without the OMI is calculated for the magnetic exponential lens model. The equation of the paraxial trajectory in fixed coordinates is directly derived from the equation of general trajectory. However, in the DA technique, the trajectory equation in rotating coordinates becomes most important because it makes the DA description very easy and straight forward to expand to higher-order aberrations. Table 1 represents the results of the magnetic exponential model lens. The optical properties are calculated by two methods: the DA and the analytical methods under the two parameters (1) $d = 0.1$ mm, and (2) $B_o = 0.01$ T. One can notice from this table that the results are very precise with a very small relative error of order (10^{-9} to 10^{-11}) and agreement between the two methods are very good. Table 2 shows the coefficients of third-order aberration as a function of the half-width (d) for the magnetic exponential model lens. The variation of the geometric aberration coefficients with d is shown in Fig. 2. From the table and the figure, one can see that astigmatism, field curvature, and the distortion (symbols C, D, and E respectively) are decreasing with increasing d under a constant value of B_o (0.1 T). This is because of the nature of these aberrations and the effect of the magnetic field distribution as shown in Fig. 1. However, the Coma (symbol F) is constant and the spherical

Table 1
Optical properties for magnetic exponential model lens calculated by DA and analytical methods under the conditions $d = 0.1$ mm and $B_o = 0.01$ T.

	$-1/f_i$	M	M_s
Analytic method	−3.176108987312	1	1
DA method	−3.176108987274	0.999999991455	1.000000000854
Relative error	-3.8×10^{-11}	8.545×10^{-9}	-8.54×10^{-9}

Table 2
Coefficients of the 3rd order aberration for various half-width (d) under $B_o = 0.01$ T.

d(m) × 10^{-3}	B (m)	F	C (m^{-1})	D (m^{-1})	E (m^{-2})
0.1	0.268931 10^{-2}	0.384567 10^{-3}	0.188722 10^{-2}	0.283038 10^{-2}	0.313290 10^{-3}
0.09	0.242038 10^{-2}	0.384567 10^{-3}	0.209691 10^{-2}	0.314487 10^{-2}	0.386775 10^{-3}
0.08	0.215145 10^{-2}	0.384566 10^{-3}	0.235903 10^{-2}	0.353798 10^{-2}	0.489510 10^{-3}
0.07	0.188251 10^{-2}	0.384563 10^{-3}	0.269603 10^{-2}	0.404341 10^{-2}	0.639360 10^{-3}
0.06	0.161358 10^{-2}	0.384561 10^{-3}	0.314537 10^{-2}	0.471731 10^{-2}	0.870239 10^{-3}
0.05	0.134465 10^{-2}	0.384558 10^{-3}	0.277444 10^{-2}	0.566077 10^{-2}	0.125315 10^{-3}

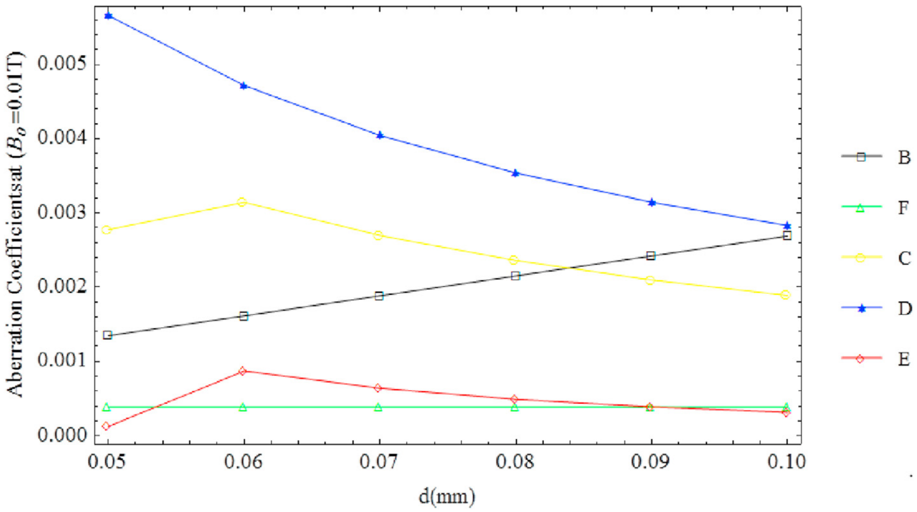

Fig. 2. Various geometrical aberration coefficients as a function of different values of half-width (d) under the maximum field $B_o = 0.01$ T.

Table 3
The 3rd order aberration coefficients for various maximum field B_o under d = 0.1 mm.

B_o(T)	B (m)	F	C (m^{-1})	D (m^{-1})	E (m^{-2})
0.01	0.268931 10^{-2}	0.384566 10^{-3}	0.188722 10^{-2}	0.283038 10^{-2}	0.313290 10^{-3}
0.05	0.674845 10^{-2}	0.240756 10^{-3}	0.473470 10^{-2}	0.707430 10^{-1}	0.195159 10^{-2}
0.1	0.273104 10^{-1}	0.387239 10^{-2}	0.191355 10^{-1}	0.282775	0.309090 10^{-1}
0.15	0.626512 10^{-1}	0.197778 10^{-1}	0.437067 10^{-1}	0.635592	0.153899
0.2	0.114432	0.632957 10^{-1}	0.789980 10^{-1}	1.12864	0.475585
0.25	0.385096	0.257083	0.125153	1.26178	1.12951

aberration coefficient (symbol B) is decreasing with decreasing d, this is because of the spherical aberration dependent on the half-width of the magnetic field distribution which is decreasing with d as shown in Fig. 1.

Table 3 and Fig. 3 show the results of the DA for the coefficients of third-order aberrations under various maximum field B_o for the magnetic exponential lens model under $d = 0.1$ mm. It is obvious that the geometrical coefficients of third-order aberration increase as the values of B_o rises. Plus, the distortion and field curvature aberrations coefficients are more affected with the increasing of B_o. Fig. 4 shows the relationship between spherical aberration disc $\mathbf{\Delta r}$, calculated using equation (16), and the $(\alpha o)^3$ under fixed magnification value $M = 0.9999$ for magnetic exponential lens model. This figure shows the disc of spherical aberration has acceptable values for $(\alpha o)^3$ values up to less than 0.2 rad.

The results of geometrical aberration coefficients of third-order (isotropic and anisotropic) using the DA method for the magnetic exponential model with and

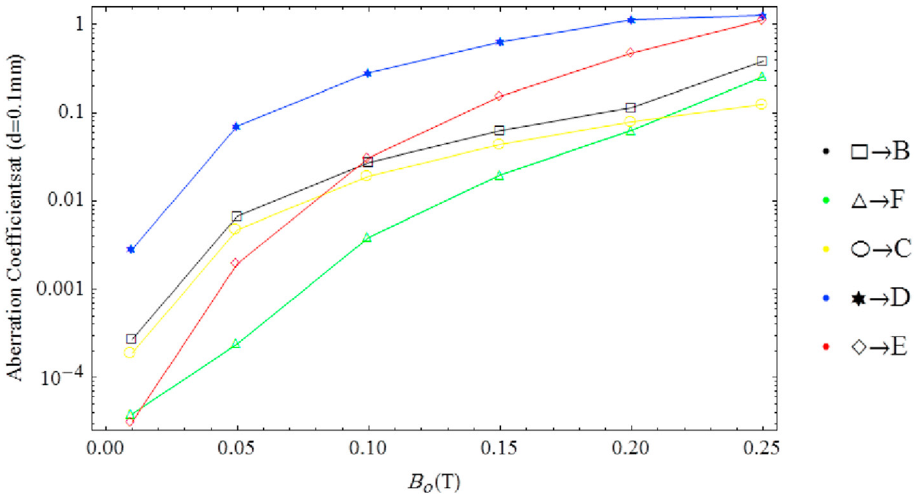

Fig. 3. Various geometrical aberration coefficients under different values of B_o for $d = 0.1$ mm.

without the OMI effect for the exact conditions used in Table 1 are shown in Table 4. The DA and the analytical methods result are shown in Tables 5−8, for the coefficients fifth-order geometrical aberration (isotropic and anisotropic) for the magnetic lens exponential model with and without OMI effect for the exact conditions in Table 1 respectively. Table 4, proves that the values of third-order spherical aberration and coma are equal as listed in equation (6). For the fifth-order spherical coefficients for isotropic and anisotropic are equal as in Tables 5 and 6 according to equations (7) and (8), respectively. The other coefficients which are listed in Tables 4−8 are computed with high precision by two methods for cross-checking. The first method is the DA method calculated using the COSYINFINITY 10 and the second method is the aberration integral method [16,25] computed using the Mathematica 9 program. It is confident that the two methods are in good agreement with a very small error of order (10^{-10}-10^{-12}). The present research proved that the technique using the DA method with COSYINFINITY 10 is an excellent tool for such calculations, and it is very compendious,

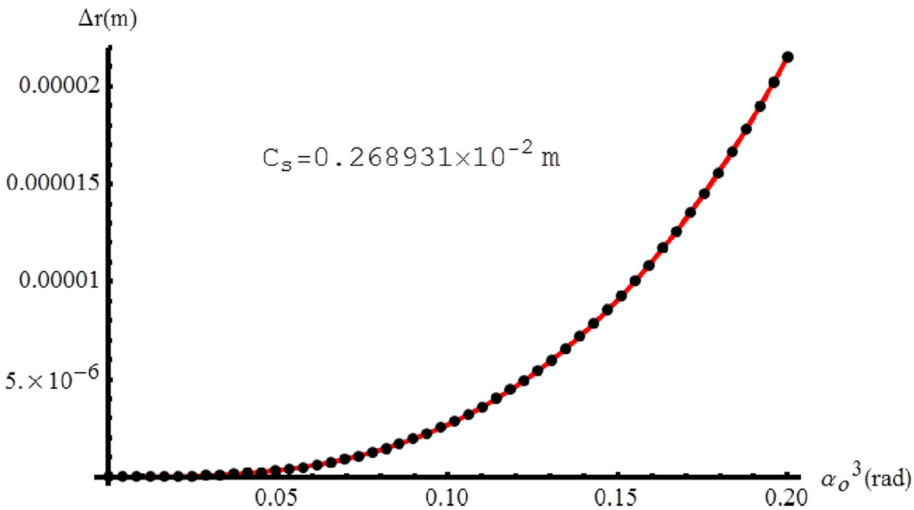

Fig. 4. Represent the relation between $(\alpha o)^3$ and the disc Δr under the magnification $M = 0.9999$ for a magnetic exponential model lens.

Table 4
The 3rd order coefficients of aberration with and without inclusion OMI effect.

Coefficient	DA Method	Aberration Integral	Relative error
B (m)	0.268931339128 10^{-2}	0.268931339126 10^{-2}	7.43687 10^{-12}
F	0.38456783501 10^{-3}	0.38456783500 10^{-3}	2.60033 10^{-11}
C (m^{-1})	0.18872246071 10^{-2}	0.18872246070 10^{-2}	5.29878 10^{-11}
D (m^{-1})	0.283038717428 10^{-2}	0.283038717427 10^{-2}	3.53317 10^{-12}
E (m^{-2})	0.31329007080 10^{-3}	0.31329007078 10^{-3}	6.38386 10^{-11}
F	0.637803310005 10^{-5}	0.637803310002 10^{-5}	4.70368 10^{-12}
c (m^{-1})	0.198149596904 10^{-1}	0.198149596901 10^{-1}	1.51401 10^{-11}
e (m^{-2})	0.249950926993 10^{-2}	0.249950926990 10^{-2}	1.20025 10^{-11}
B_m (m)	0.268931339128 10^{-2}	0.268931339126 10^{-2}	7.43687 10^{-12}
F_m	0.38456783501 10^{-3}	0.38456783500 10^{-3}	2.60033 10^{-11}
C_m (m^{-1})	0.18872134623 10^{-2}	0.18872134621 10^{-2}	1.05976 10^{-10}
D_m (m^{-1})	0.283038234126 10^{-2}	0.283038234122 10^{-2}	1.41323 10^{-11}
E_m (m^{-2})	0.31328865412 10^{-3}	0.31328865409 10^{-3}	9.57585 10^{-11}
f_m	0.117852587460 10^{-4}	0.117852587458 10^{-4}	1.69705 10^{-11}
c_m (m^{-1})	0.198136696323 10^{-1}	0.198136696321 10^{-1}	1.00939 10^{-11}
e_m (m^{-2})	0.194594999224 10^{-2}	0.194594999220 10^{-2}	2.05555 10^{-11}

Table 5
The 5th order isotropic aberration coefficients without inclusion OMI.

Coefficient	DA Method	Aberration Integral	Relative error
A_5 (m)	0.931253414197 10^{-2}	0.931253414197 10^{-2}	0
B_{51}	0.288216273304 10^{-2}	0.288216273301 10^{-2}	1.0409 10^{-11}
B_{52}	0.182823992662 10^{-2}	0.182823992660 10^{-2}	1.09395 10^{-11}
C_{51} (m^{-1})	0.141807872152	0.141807872149	2.11553 10^{-11}
C_{52} (m^{-1})	0.118681906042 10^{1}	0.118681906042 10^{1}	0
C_{53} (m^{-1})	0.141287570572	0.141287570570	1.41556 10^{-11}
D_{51} (m^{-2})	0.442053911493	0.442053911492	2.26212 10^{-12}
D_{52} (m^{-2})	0.493036611338 10^{-1}	0.493036611338 10^{-1}	0
D_{53} (m^{-2})	0.155757572463	0.155757572463	0
E_{51} (m^{-3})	0.296971216887 10^{1}	0.296971216884 10^{1}	1.0102 10^{-11}
E_{52} (m^{-3})	0.398051995876 10^{2}	0.398051995873 10^{2}	7.53666 10^{-12}
F_5 (m^{-4})	0.156360654477 10^{2}	0.156360654475 10^{2}	1.27909 10^{-11}

Table 6
The 5th order isotropic aberration coefficients with inclusion OMI.

Coefficient	DA Method	Aberration Integral	Relative error
A_{5m} (m)	0.931253414197 10^{-2}	0.931253414197 10^{-2}	0
B_{51m}	0.288214942464 10^{-2}	0.288214942461 10^{-2}	1.04089 10^{-11}
B_{52m}	0.182825057334 10^{-2}	0.182825057332 10^{-2}	1.09395 10^{-11}
C_{51m} (m^{-1})	0.141248593697	0.141248593696	7.07956 10^{-12}
C_{52m} (m^{-1})	0.118681963757 10^{1}	0.118681963755 10^{1}	1.68518 10^{-11}
C_{53m} (m^{-1})	0.141287952144	0.141287952141	2.12332 10^{-11}
D_{51m} (m^{-2})	0.441881332967	0.441881332967	0
D_{52m} (m^{-2})	0.496483690697 10^{-1}	0.496483690694 10^{-1}	6.04242 10^{-12}
D_{53m} (m^{-2})	0.155527900500	0.155527900495	3.21486 10^{-11}
E_{51m} (m^{-3})	0.296966329314 10^{1}	0.296966329311 10^{1}	1.01022 10^{-11}
E_{52m} (m^{-3})	0.398051939731 10^{2}	0.398051939727 10^{2}	1.00489 10^{-11}
F_{5m} (m^{-4})	0.156302530909 10^{2}	0.156302530906 10^{2}	1.91935 10^{-11}

Table 7
The 5th order anisotropic aberration coefficients without inclusion OMI.

Coefficient	DA Method	Aberration Integral	Relative error
a_5 (m)	$0.229990514369\ 10^{-5}$	$0.229990514369\ 10^{-5}$	0
b_{51}	$0.979213080534\ 10^{-4}$	$0.979213080531\ 10^{-4}$	$3.06381\ 10^{-12}$
b_{52}	$0.186258538323\ 10^{-3}$	$0.186258538320\ 10^{-3}$	$1.61066\ 10^{-11}$
c_{51} (m^{-1})	0.198652606142	0.198652606140	$1.00679\ 10^{-11}$
c_{52} (m^{-1})	$0.497138529482\ 10^{-1}$	$0.497138529480\ 10^{-1}$	$4.02302\ 10^{-12}$
c_{53} (m^{-1})	$0.994282614349\ 10^{-1}$	$0.994282614346\ 10^{-1}$	$3.01735\ 10^{-12}$
d_{51} (m^{-2})	$0.447156881002\ 10^{-2}$	$0.447156881000\ 10^{-2}$	$4.47281\ 10^{-12}$
d_{52} (m^{-2})	$0.304936612769\ 10^{-1}$	$0.304936612768\ 10^{-1}$	$3.27948\ 10^{-12}$
d_{53} (m^{-2})	$0.486108121043\ 10^{-1}$	$0.486108121042\ 10^{-1}$	$2.05708\ 10^{-12}$
e_{51} (m^{-3})	$0.100331064244\ 10^{2}$	$0.100331064241\ 10^{2}$	$2.9901\ 10^{-11}$
e_{52} (m^{-3})	$0.502285030465\ 10^{1}$	$0.502285030463\ 10^{1}$	$3.9818\ 10^{-12}$
f_5 (m^{-4})	0.997977018525	0.997977018524	$1.002\ 10^{-12}$

Table 8
The 5th order anisotropic aberration coefficients with inclusion OMI.

Coefficient	DA Method	Aberration Integral	Relative error
a_{5m} (m)	$0.229990514369\ 10^{-5}$	$0.229990514369\ 10^{-5}$	0
b_{51m}	$0.151808286866\ 10^{-3}$	$0.151808286864\ 10^{-3}$	$1.31746\ 10^{-11}$
b_{52m}	$0.143148955273\ 10^{-3}$	$0.143148955271\ 10^{-3}$	$1.39715\ 10^{-11}$
c_{51m} (m^{-1})	0.204123983826	0.204123983825	$4.89901\ 10^{-12}$
c_{52m} (m^{-1})	$0.497005108710\ 10^{-1}$	$0.497005108707\ 10^{-1}$	$6.03608\ 10^{-12}$
c_{53m} (m^{-1})	$0.994240297786\ 10^{-1}$	$0.994240297785\ 10^{-1}$	$1.00569\ 10^{-12}$
d_{51m} (m^{-2})	$0.351050788405\ 10^{-3}$	$0.351050788402\ 10^{-3}$	$8.54574\ 10^{-12}$
d_{52m} (m^{-2})	$0.300019222401\ 10^{-1}$	$0.300019222398\ 10^{-1}$	$9.99935\ 10^{-12}$
d_{53m} (m^{-2})	$0.479555600780\ 10^{-1}$	$0.479555600777\ 10^{-1}$	$6.25586\ 10^{-12}$
e_{51m} (m^{-3})	$0.100319033847\ 10^{2}$	$0.100319033845\ 10^{2}$	$1.99364\ 10^{-11}$
e_{52m} (m^{-3})	$0.502182732501\ 10^{1}$	$0.502182732500\ 10^{1}$	$1.99113\ 10^{-12}$
f_{5m} (m^{-4})	0.951910465150	0.951910465147	$3.15164\ 10^{-12}$

effective, and accurate for high order aberration analysis.

Acknowledgements

Special appreciation to Professor M. Berz for providing COSY INFINITY 10 licenses for the present work. Our great appreciations and thanks to Professor Zhixiong Liu for encouragement and consultations through our work. Also, our appreciations to Professor Kyoko Makino for important and valuable notes about COSY INFINITY 10.

References

[1] H. Rose, W. Wan, Aberration correction in electron microscopy, in: IEEE Proceedings of Particle Accelerator Conference, Knoxville, Tennessee, USA, 2005, pp. 44−48.

[2] M. Haider, P. Hartel, H. Muler, S. Uhemann, J. Zach, Current and future aberration correctors for the improvement of resolution in electron microscopy, Phil. Trans. R. Soc. A 367 (2009) 3665−3682.

[3] P.W. Hawkes, The correction of electron lens aberrations, Ultramicroscopy 156 (2015) 1−64, https://doi.org/10.1016/j.ultramic.2015.03.007.

[4] M. Berz, Differential algebraic description of beam dynamics to very high orders, Part. Accel. 24 (1989) 109−124.

[5] L.P. Wang, T.T. Tang, B.J. Cheng, J. Cai, Automatic differentiation method for the aberration analysis of electron optical systems, Optik 110 (1999) 408−414.

[6] Y. Kang, T. Tang, Y. Ren, X. Guo, Differential algebraic method for computing the high order aberrations of practical electron lenses, Optik 118 (2007) 158−162.

[7] Z. Liu, The Seventh International Computational Accelerator Physics, Michigan State University, East Lansing, MI 48824, USA, 2002, pp. 185−192. October 22−25.

[8] Y. Kang, T. Tang, J. Zhao, S. Li, D. Zhang, A different algebraic method for the fifth-order combined geometric−chromatic aberrations of practical magnetic electron lenses, Optik 121 (2010) 178−183.

[9] J. Ximen, Z. Liu, Third-order geometric aberrations in Glaser's bell-shaped magnetic lens for object magnetic immersion, Optik 111 (2000) 355−358.

[10] O. Seman, The Theoretical Basis of Electron Optics, Higher Education Press, Beijing, 1958.

[11] M. Berz, Kyoko Makino, COSY INFINITY 10 Programmer's Manual, MSU Report MSUHEP, October 2017, p. 151102.

[12] M. Berz, Kyoko Makino, COSY INFINITY 10 Beam Physics Manual, MSU Report MSUHEP, October 2017, p. 151103.

[13] S. Wolfram, The MATHEMATICA Book, fifth ed., Wolfram Media, Illinois, 2003.

[14] Z. Liu, Differential algebraic description for third- and fifth-order aberrations of electromagnetic lenses, Nucl. Instrum. Methods A 519 (2004) 154−161.

[15] A. Amer, Electron Lenses Chromatic and Geometrical Aberration Coefficients Calculations by Using Differential Algebraic Method, Master Thesis, Al-Nahrain University, 2018.

[16] J. Ximen, Aberration theory in electron and ion optics, in: Adv. In Electronics and Electron Phys., Supplement 17, Academic Press, New York, 1986.

[17] J. Orloff, Handbook of Charged Particle Optics, second ed., 2009, p. 210, ch.6.

[18] A. Amer, A.K. Ahmad, Differential algebraic description for aberrations analysis of typical electrostatic einzel lens, Optik 168 (2018) 112−117.

[19] Z. Liu, Differential algebraic method for aberration analysis of typical electrostatic lenses, Ultramicroscopy 106 (2006) 220−232.

[20] P.W. Hawkes, The geometrical aberration of general electron optical system? II. The primary (third order) aberration of orthogonal system, and the secondary (fifth order) aberration of round system, Phil. Trans. 257 (1965) 523.

[21] J. Ximen, J. Liang, Z. Liu, Analytical analysis and numerical calculation of third-order and fifth-order aberrations in Glaser's bell-shaped magnetic lens, Optik 102 (1996) 24−30.

[22] M. Berz, Modern map methods in particle beam physics, Adv. Imag. Electron. Phys. 108 (1999) 1−138.

[23] T. Radlicka, Lie algebraic methods in charged particle optics, in: P. Hawkes (Ed.), Adv. In Electronics and Electron Phys. vol. 151, Academic Press, New York, 2008, p. 242.

[24] J. Ximen, Z. Liu, A theorem of higher-order spherical aberration in glasers bell-shaped magnetic lens, Optik 107 (1) (1997) 17−25.

[25] P. Hawkes, E. Kasper, Principles of Electron Optics, second ed., vol. 1, 2018 ch. 36, AP.

[26] K. Makino, M. Berz, COSY INFINITY version 8, Nucl. Instrum. Methods A 427 (1999) 338−343.

[27] K. Makino, The COSY 8th Order Runge-Kutta Integrator, Department of Physics, University of Illinois at Urbana-Champaign, 2002. (Accessed 14 March 2002).

[28] A.K. Ahmad, S.M. Juma, Fatin A.J. Al-Mudarris, Simulation of an ion optical transport and focusing system, Iraqi J. Phy. 7 (2009) 40−46.

[29] A.K. Ahmad, F.A. Ali, A.A. Al-Tabbak, S.M. Juma, Computer aided design of a magnetic lens using a combined dynamic programming and artificial intelligence technique, Iraqi J. Appl. Phy. 10 (2014) 33−37.