Third- and Fifth-Order Geometric Aberrations in Magnetic Exponential Lens Model for Object Magnetic Immersion

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Third- and Fifth-Order Geometric Aberrations in Magnetic Exponential Lens Model for Object Magnetic Immersion

Abstract

We numerically employ the differential algebraic technique to calculate the third and fifth-order geometric aberrations coefficients, which are derived by using the map method, of the magnetic exponential lens model. These coefficients are calculated for object magnetic immersion (OMI). The magnetic exponential model is used as an example for the magnetic round lens to calculate the coefficients. The numerical electron optical results are perfectly in match with the analytically results with a very small relative error ($10^{-10}$ - $10^{-12}$). The DA advantage technique is a very compendious, effective, proper, and accurate for analysis of electron optical aberration. The COSYINFINITY 10 code is used in our calculations.

Keywords

Differential algebra, Map method, Magnetic round lens, high order Geometric aberrations, OMI effect, COSYINFINITY 10

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Cover Page Footnote

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1. Introduction

Through the growing applications of high-quality systems of focusing charged particle beams such as high-resolution electron microscopes, electron and ion lithography systems, and beam accelerators, the desire to design practical electromagnetic lenses with aberrations of higher orders than third-order is receiving international attention [1–3]. Therefore, it is required to study the higher-order aberration for such focusing systems. Differential algebra (DA) is a highly accurate and advanced tool for solving sets of nonlinear dynamical equations as in the high-order aberrations of systems of electron-optical lenses. The DA method is a very accurate method with no special requirements for the calculation, developed by many authors including Berz [4], Wang and Chen [5,6] and Liu [7]; it can be applied to problems in the electric and/or magnetic fields engineering design. In this work, we utilize the DA method and COSY INFINITY 10 program to calculate practical electron lenses with magnetic fields that include object magnetic immersion (OMI) corrections. The OMI correction method can be used to correct chromatic and spherical aberration coefficients for magnetic lenses when the object is located in the field of the magnetic lens. The Gaussian properties and the third and fifth orders aberrations are determined for an exponential magnetic electron lens whose magnetic field magnitudes are input into a discretized array structure for further analysis. Three DA variable types exist for analyzing the aberration of rotationally symmetric electron optical systems: the fixed coordinates DA description, the rotating coordinates DA description, and the hybrid coordinates DA description [7,8]. The rotational coordinate system is used to facilitate the analytical paraxial ray equation. In the magnetic lens case, if the object immersed in the field, the correction of aberration done using OMI effect [9]. In the case of higher-order aberration (Fifth and Seventh), the description of DA in rotational coordinates becomes very complicated. We derive the general equation of electron trajectory and we obtain the transfer map of the rotational coordinates by tracking the trajectory equation. Seman [10] was the first researcher who discussed the problems of object magnetic immersion OMI in magnetic lenses. The high order aberrations (third and fifth) for the magnetic exponential model are carried out using the DA technique. In this work, the DA technique using COSY INFINITY 10 [11,12] is used for such calculations for the first time. The DA results were crosschecked using the high order (third and fifth) aberration integrals evaluated by Wolfram Mathematica 9 program [13]. The results proved to be very compendious, effective, and accurate. The expression of the general equation of electron trajectory in rotating and fixed coordinates are given in many references [14–19]. It performs a significant part in clarifying the differential algebraic method in rotational coordinates.

2. DA description types of high order geometrical aberrations

Tracking the equation of the general trajectory of the electrons is used in fixed coordinates to obtain the higher orders (third and fifth) geometric aberrations using the deferential algebra integrator limit starting at 

\[ z_o (\text{plane of object}) \rightarrow z_i (\text{plane of image}) \] 

in fixed coordinate [18]:

\[
\Delta X_{klm} = \sum_{k+l+m+n=3}^{k+l+m+n=3} M_f (1.klmn) X^x_{o} X^{y'}_{o} Y^m_{o} Y^n_{o},
\]

\[
\Delta Y_{klm} = \sum_{k+l+m+n=3}^{k+l+m+n=3} M_f (1.klmn) X^y_{o} X^{y'}_{o} Y^m_{o} Y^n_{o}, \quad (1)
\]

\[
\Delta X_{k+1+m+n=5} = \sum_{k+l+m+n=5}^{k+l+m+n=5} M_f (1.klmn) X^x_{o} X^{y'}_{o} Y^m_{o} Y^n_{o},
\]

\[
\Delta Y_{k+1+m+n=5} = \sum_{k+l+m+n=5}^{k+l+m+n=5} M_f (3.klmn) X^y_{o} X^{y'}_{o} Y^m_{o} Y^n_{o},
\]

where \( M_f (i, klmn) \) is the transfer map elements in fixed coordinates, the number of orders depending on the sum of \( k, l, m, \) and \( n \). While tracking the expression of rotating transform coordinates the DA characterization of rotational coordinates are calculated [16,18] as in the following form:

\[
\Delta x_{klm} = \sum_{k+l+m+n=3}^{k+l+m+n=3} M_r (1.klmn) x^{x'}_{o} x^{y'}_{o} y^{m}_{o} y^{n}_{o},
\]

\[
\Delta y_{klm} = \sum_{k+l+m+n=3}^{k+l+m+n=3} M_r (3.klmn) x^{x'}_{o} x^{y'}_{o} y^{m}_{o} y^{n}_{o}, \quad (2)
\]

\[
\Delta x_{klm} = \sum_{k+l+m+n=5}^{k+l+m+n=5} M_r (1.klmn) x^{x'}_{o} x^{y'}_{o} y^{m}_{o} y^{n}_{o},
\]

\[
\Delta y_{klm} = \sum_{k+l+m+n=5}^{k+l+m+n=5} M_r (1.klmn) x^{x'}_{o} x^{y'}_{o} y^{m}_{o} y^{n}_{o},
\]

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where \( M_i \) (i, klmn) is the transfer map elements in the rotating coordinates. The third-order geometrical aberrations have an expansion, in the vector form, of combined electromagnetic lenses

\[
\Delta \mathbf{r}_3 = B \left( \mathbf{r}_o - \mathbf{r}_o' \right) + F \left( \mathbf{r}_o - \mathbf{r}_o' \right) \mathbf{r}_o + 2 \left( \mathbf{r}_o - \mathbf{r}_o' \right) \mathbf{r}_o' + 2C \left( \mathbf{r}_o - \mathbf{r}_o' \right) \mathbf{r}_o + D \left( \mathbf{r}_o - \mathbf{r}_o' \right) \mathbf{r}_o' + E \left( \mathbf{r}_o - \mathbf{r}_o' \right) \mathbf{r}_o + \cdots
\]

and the fifth order coefficients are,

\[
A_5 = M_r(1.0500), B_{51} = \frac{M_r(1.1004)}{2}, B_{52} = \frac{M_r(1.0902)}{3}, C_{51} = M_r(1.1011)/2, C_{52} = M_r(1.1002),
\]

\[
D = M_r(1.0120), E = M_r(1.3000), f = M_r(3.1002)/3, c = M_r(3.1011)/2, e = M_r(3.3000),
\]

In addition, the expansion, for combined electromagnetic lenses, of the fifth order geometrical aberrations in a vector form is;

\[
\Delta \mathbf{r}_5 = \left( \mathbf{r}_o - \mathbf{r}_o' \right) \left( A_5 \mathbf{r}_o + a_5 \mathbf{r}_o' \right) + \left( \mathbf{r}_o - \mathbf{r}_o' \right) \left( B_{51} \mathbf{r}_o + b_{51} \mathbf{r}_o' \right) + \cdots
\]

In comparison equation (2) with equations (3) and (4), the connection between the elements of the map in Eq. (2) and the coefficients of aberration in equations (3) and (4) are obtained. The Glaser's notation of the third order coefficients of the geometrical aberration

\[
\mathbf{r}_o = (x_o, y_o), \quad \mathbf{r}_o' = (x_o', y_o'), \quad \mathbf{r}_o^* = (y_o, -x_o)
\]
where the suffix "m" indicates using the effect of OMI, the aberrations were corrected. The hybrid coordinates transfer map elements are represented by $M_h(1.klmm)$ and $M_h(3.klmm)$, where they could be derived from $M_f(1.klmm)$ and $M_f(3.klmm)$. The relation between the geometrical aberration coefficients and the hybrid coordinates map elements together with the containment of the effect of OMI is calculated. The aberration coefficients of the third order isotropic are:

$$B_m = M_h(1.0300) = M_f(3.0300)\sin \theta_1 + M_f(1.0300)\cos \theta_1$$
$$F_m = M_h(1.1002) = M_f(3.1002)\sin \theta_1 + M_f(1.1002)\cos \theta_1$$
$$C_m = M_h(1.1011) = \left[ M_f(3.1011)\sin \theta_1 + M_f(1.1011)\cos \theta_1 \right] / 2$$
$$D_m = M_h(1.0120) = M_f(3.0120)\sin \theta_1 + M_f(1.0120)\cos \theta_1$$
$$E_m = M_h(1.3000) = M_f(3.3000)\sin \theta_1 + M_f(1.3000)\cos \theta_1$$

(6)

The aberration coefficients of the third order anisotropic are:

$$f_m = M_h(3.1002) = \left[ -M_f(1.1002)\sin \theta_1 + M_f(3.1002)\cos \theta_1 \right] / 3$$
$$c_m = M_h(3.1011) = \left[ -M_f(1.1011)\sin \theta_1 + M_f(3.1011)\cos \theta_1 \right] / 2$$
$$e_m = M_h(3.3000) = \left[ -M_f(1.3000)\sin \theta_1 + M_f(3.3000)\cos \theta_1 \right]$$

(7)

The aberration coefficients of the fifth order isotropic are:

$$A_{5m} = M_h(1.0500) = M_f(3.0500)\sin \theta_1 + M_f(1.0500)\cos \theta_1$$
$$B_{51m} = M_h(1.1004) = M_f(3.1004)\sin \theta_1 + M_f(1.1004)\cos \theta_1$$
$$B_{52m} = M_h(1.0311) = M_f(3.0311)\sin \theta_1 + M_f(1.0311)\cos \theta_1$$
$$C_{51m} = M_h(1.1013) = M_f(3.1013)\sin \theta_1 + M_f(1.1013)\cos \theta_1$$
$$C_{52m} = M_h(1.0320) = M_f(3.0320)\sin \theta_1 + M_f(1.0320)\cos \theta_1$$
$$C_{53m} = M_h(1.0122) - C_{52m} = M_f(3.0122)\sin \theta_1 + M_f(1.0122)\cos \theta_1 - C_{52m}$$
$$D_{51m} = M_h(1.3002) = M_f(3.3002)\sin \theta_1 + M_f(1.3002)\cos \theta_1$$
$$D_{52m} = M_h(1.1022) - D_{51m} = M_f(3.1022)\sin \theta_1 + M_f(1.1022)\cos \theta_1 - D_{51m}$$
$$D_{53m} = M_h(1.0131) = M_f(3.0131)\sin \theta_1 + M_f(1.0131)\cos \theta_1$$
$$E_{51m} = M_h(1.3011) = M_f(3.3011)\sin \theta_1 + M_f(1.3011)\cos \theta_1$$
$$E_{52m} = M_h(1.0140) = M_f(3.0140)\sin \theta_1 + M_f(1.0140)\cos \theta_1$$
$$E_{53m} = M_h(1.5000) = M_f(3.5000)\sin \theta_1 + M_f(1.5000)\cos \theta_1$$

(8)

The aberration coefficients of the fifth order anisotropic are:

$$a_{5m} = M_h(3.0500) = -M_f(1.0500)\sin \theta_1 + M_f(3.0500)\cos \theta_1$$
$$b_{52m} = M_h(3.0311) = -M_f(1.0311)\sin \theta_1 + M_f(3.0311)\cos \theta_1$$
$$c_{51m} = M_h(3.1013) = -M_f(1.1013)\sin \theta_1 + M_f(3.1013)\cos \theta_1$$
$$c_{52m} = M_h(3.0320) = -M_f(1.0320)\sin \theta_1 + M_f(3.0320)\cos \theta_1$$
$$c_{53m} = M_h(3.0122) - c_{52m} = -M_f(1.0122)\sin \theta_1 + M_f(3.0122)\cos \theta_1 - c_{52m}$$
$$d_{51m} = M_h(3.3002) = -M_f(1.3002)\sin \theta_1 + M_f(3.3002)\cos \theta_1$$
$$d_{52m} = M_h(1.1022) - d_{51m} = -M_f(1.1022)\sin \theta_1 + M_f(3.1022)\cos \theta_1 - d_{51m}$$
$$d_{53m} = M_h(3.0131) = -M_f(1.0131)\sin \theta_1 + M_f(3.0131)\cos \theta_1$$
$$e_{51m} = M_h(3.3011) = -M_f(1.3011)\sin \theta_1 + M_f(3.3011)\cos \theta_1$$
$$e_{52m} = M_h(3.0140) = -M_f(1.0140)\sin \theta_1 + M_f(3.0140)\cos \theta_1$$
$$f_{5m} = M_h(3.5000) = -M_f(1.5000)\sin \theta_1 + M_f(3.5000)\cos \theta_1$$

(9)
3. High order aberrations of the magnetic lens

The concept of higher-order aberration coefficients derivation is the same as that in Refs. [18,19]. According to Equation (3), there are three anisotropic (lowercase letters) and five isotropic (uppercase letters) coefficients of the third-order aberration: the symbol B refers to the spherical aberration, the symbols F and f refers to coma, the symbols C and c refers to astigmatism, the symbol D refers to the field curvature, and the symbols E and e refers to the distortion [20–22]. The high order coefficients formulas for OMI correction are given as follows:

\[
\begin{align*}
B_m &= B, \quad F_m = F, \quad C_m &= C + 2\theta_o f + \theta_o^2 B, \\
D_m &= D - 6\theta_o f + 3\theta_o^2 B, \quad E_m = E - 2\theta_o c + \theta_o^2 F, \\
f_m &= f - \theta_o B, \quad c_m = c - \theta_o F. \quad e_m = e - \theta_o D + 3\theta_o^2 f - \theta_o^3 B
\end{align*}
\]

\[
\begin{align*}
A_{5m} &= A_5, \quad B_{51m} &= B_{51} + 5\theta_o \dot{a}_5, \\
B_{52m} &= B_{52} - 4\theta_o \dot{a}_5, \\
C_{51m} &= C_{51} + \theta_o \dot{a}_5 (4b_{51} + 3b_{52}) - 4\theta_o^2 A_5, \\
C_{52m} &= C_{52} - 4\theta_o \dot{a}_5 b_{51} + 10\theta_o^2 A_5, \\
C_{53m} &= C_{53} - 2\theta_o \dot{a}_5 b_{52} - 4\theta_o^2 A_5, \\
D_{51m} &= D_{51} + 3\theta_o \dot{a}_5 c_{52} + 6\theta_o^2 b_{51} + 10\theta_o^3 A_5, \\
D_{52m} &= D_{52} + \theta_o \dot{a}_5 (2c_{51} + 3c_{52}) - 2\theta_o^2 (2B_{51} + B_{52}) - 4\theta_o^3 A_5, \\
D_{53m} &= D_{53} - 2\theta_o \dot{a}_5 (c_{51} + c_{52}) - 3\theta_o^2 B_{52} - 4\theta_o^3 A_5, \\
E_{51m} &= E_{51} + \theta_o \dot{a}_5 (2d_{51} + d_{52}) + \theta_o^2 (C_{51} - 2C_{52}) + \theta_o^3 (4b_{51} + b_{52}) - 4\theta_o^4 A_5, \\
E_{52m} &= E_{52} - 2\theta_o \dot{a}_5 d_{51} + 3\theta_o^2 c_{52} - 4\theta_o^3 b_{51} + 5\theta_o^4 A_5, \\
F_{5m} &= F_5 + \theta_o \dot{e}_5 + \theta_o^2 D_{51} + \theta_o^3 c_{52} + \theta_o^4 B_{51} + \theta_o^5 a_5; \\
da_{5m} &= a_5, \quad b_{51m} = b_{51} - 5\theta_o \dot{a}_5 A_5, \quad b_{52m} = b_{52} + 4\theta_o \dot{a}_5 A_5, \\
c_{51m} &= c_{51} - \theta_o \dot{a}_5 (4B_{51} + 3B_{52}) - 4\theta_o^2 a_5, \\
c_{52m} &= c_{52} + 4\theta_o \dot{a}_5 B_{51} + 10\theta_o^2 a_5, \\
c_{53m} &= c_{53} + 2\theta_o \dot{a}_5 B_{52} - 4\theta_o^2 a_5, \\
d_{51m} &= d_{51} - 3\theta_o \dot{a}_5 C_{52} + 6\theta_o^2 b_{51} - 10\theta_o^3 A_5, \\
d_{52m} &= d_{52} + \theta_o \dot{a}_5 (2C_{51} + C_{52}) - 2\theta_o^2 (2b_{51} + b_{52}) + 4\theta_o^3 A_5, \\
d_{53m} &= d_{53} + 2\theta_o \dot{a}_5 (C_{51} + C_{52}) + 3\theta_o^2 b_{52} + 4\theta_o^3 A_5, \\
e_{51m} &= e_{51} - \theta_o \dot{a}_5 (2D_{51} + D_{52}) + \theta_o^2 (c_{51} - 2c_{52}) + \theta_o^3 (4B_{51} + B_{52}) - 4\theta_o^4 a_5, \\
e_{52m} &= e_{52} + 2\theta_o \dot{a}_5 D_{51} + 3\theta_o^2 c_{52} + 4\theta_o^3 B_{51} + 5\theta_o^4 a_5, \\
f_{5m} &= f_5 - \theta_o \dot{e}_5 E_{52} + \theta_o^2 d_{51} - \theta_o^3 C_{52} + \theta_o^4 b_{51} - \theta_o^5 A_5,
\end{align*}
\]

where \(\theta_o := \frac{\eta B(z_o)}{V(z_o)}\), \(\eta := \sqrt{2m}\), \(B(z_o)\) and \(V(z_o)\) are the magnetic induction and potential function respectively at the object plane \(z_o\).

4. Computational illustration

The former derivations were a concise description of the basics of the high order geometric aberrations calculation for electron lenses with magnetic fields that
include OMI corrections. More information about the DA method can be found in Berz (1999) [22]. The formulation of the DA method is independent of the aberration order, thus, to understand the form of the Hamiltonian an analytical description of the field must be defined. Thus, we must know the exact form of the axial potentials and their derivatives, to determine any order of aberrations. Therefore, the inaccuracy of the axial potential higher-order derivatives is a limit for the use of the DA method in real optical systems, i.e. the analytical field model (or the axil potential) must have a trajectory equation that can be solved analytically and its higher order aberrations can also be expressed by the analytical expression of the field model to have an accurate solution [23,24].

In the present work, the exponential model of the magnetic round lens is one of the well-known models that can make the paraxial ray equation soluble and useful, especially for ‘single-pole’ lenses [25], whose magnetic induction distributions have the following form

\[ B(z) = B_0 e^{-2z^2} \]  

(15)

where \( B_0 = 0.01 \) T is the maximum magnetic flux and \( d \) is the half-width of the magnetic flux distribution in mm. The axial magnetic flux distributions are \( B(z) \) shown in Fig. 1, along \( z \) (optical axis) from \( z_o = -0.3 \) mm (object plane) to \( z_i = 0.3 \) mm (image plane) at different values of \( d \) (\( d = 0.1, 0.09, 0.08, 0.07, 0.06, 0.05 \) mm). Two packages are used: the COSY INFINITY10 [11,12,26] and Wolfram Mathematica version 9 [13]. With the aid of the 8th order Runge-Kutta integrator [27], we calculate the transfer map produced by tracking the trajectory equations in rotating and fixed coordinates. All the high order aberration coefficients (third and fifth) are calculated using two packages (without and with the OMI participation) for the magnetic exponential model. The spherical aberration disc \( \Delta r \) (disc of least confusion) is also calculated using the following equation [28,29]:

\[ \Delta r = MB\alpha_o^3. \]  

(16)

where \( M \) is the magnification, \( B \) (in some reference denoted by \( C_s \)) is the third order spherical aberration coefficient and \( \alpha_o \) is the half angle in radian and we take a range of \( (\alpha_o)^3 \) from zero to 0.2 radian.

5. Discussion and conclusion

The third- and fifth-order isotropic and anisotropic geometric aberration with and without the OMI is calculated for the magnetic exponential lens model. The equation of the paraxial trajectory in fixed coordinates is directly derived from the equation of general trajectory. However, in the DA technique, the trajectory equation in rotating coordinates becomes most important because it makes the DA description very easy and straight forward to expand to higher-order aberrations. Table 1 represents the results of the magnetic exponential model lens. The optical properties are calculated by two methods: the DA and the analytical methods under the two parameters (1) \( d = 0.1 \) mm, and (2) \( B_0 = 0.01 \) T. One can notice from this table that the results are very precise with a very small relative error of order \( 10^{-9} \) to \( 10^{-11} \) and agreement between the two methods are very good. Table 2 shows the coefficients of third-order aberration as a function of the half-width (\( d \)) for the magnetic exponential model lens. The variation of the geometric aberration coefficients with \( d \) is shown in Fig. 2. From the table and the figure, one can see that astigmatism, field curvature, and the distortion (symbols C, D, and E respectively) are decreasing with increasing \( d \) under a constant value of \( B_0 \) (0.1 T). This is because of the nature of these aberrations and the effect of the magnetic field distribution as shown in Fig. 1. However, the Coma (symbol F) is constant and the spherical aberration of the model.

<table>
<thead>
<tr>
<th>d (mm)</th>
<th>1/f_i</th>
<th>M</th>
<th>M_1</th>
</tr>
</thead>
<tbody>
<tr>
<td>Analytic method</td>
<td>-3.176108987312</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>DA method</td>
<td>-3.176108987274</td>
<td>0.999999991455</td>
<td>1.000000000854</td>
</tr>
<tr>
<td>Relative error</td>
<td>(-3.8 \times 10^{-11})</td>
<td>(8.545 \times 10^{-9})</td>
<td>(-8.54 \times 10^{-9})</td>
</tr>
</tbody>
</table>
aberration coefficient (symbol B) is decreasing with decreasing d, this is because of the spherical aberration dependent on the half-width of the magnetic field distribution which is decreasing with d as shown in Fig. 1.

Table 3 and Fig. 3 show the results of the DA for the coefficients of third-order aberrations under various maximum field \( B_0 \) for the magnetic exponential lens model under \( d = 0.1 \) mm. It is obvious that the geometrical coefficients of third-order aberration increase as the values of \( B_0 \) rises. Plus, the distortion and field curvature aberrations coefficients are more affected with the increasing of \( B_0 \). Fig. 4 shows the relationship between spherical aberration disc \( \Delta r \), calculated using equation (16), and the \((\mathbf{z} \mathbf{o})^3\) under fixed magnification value \( M = 0.9999 \) for magnetic exponential lens model. This figure shows the disc of spherical aberration has acceptable values for \((\mathbf{z} \mathbf{o})^3\) values up to less than 0.2 rad.

The results of geometrical aberration coefficients of third-order (isotropic and anisotropic) using the DA method for the magnetic exponential model with and
without the OMI effect for the exact conditions used in Table 1 are shown in Table 4. The DA and the analytical methods result are shown in Tables 5–8, for the coefficients fifth-order geometrical aberration (isotropic and anisotropic) for the magnetic lens exponential model with and without OMI effect for the exact conditions in Table 1 respectively. Table 4, proves that the values of third-order spherical aberration and coma are equal as listed in equation (6). For the fifth-order spherical coefficients for isotropic and anisotropic are equal as in Tables 5 and 6 according to equations (7) and (8), respectively. The other coefficients which are listed in Tables 4–8 are computed with high precision by two methods for cross-checking. The first method is the DA method calculated using the COSYINFINITY 10 and the second method is the aberration integral method [16,25] computed using the Mathematica 9 program. It is confident that the two methods are in good agreement with a very small error of order \(10^{-10} - 10^{-12}\). The present research proved that the technique using the DA method with COSYINFINITY 10 is an excellent tool for such calculations, and it is very compendious,
The 5th order isotropic aberration coefficients with inclusion OMI.

<table>
<thead>
<tr>
<th>Coefficient</th>
<th>DA Method</th>
<th>Aberration Integral</th>
<th>Relative error</th>
</tr>
</thead>
<tbody>
<tr>
<td>B (m)</td>
<td>0.268931339128 10^{-2}</td>
<td>0.268931339126 10^{-2}</td>
<td>7.43687 10^{-12}</td>
</tr>
<tr>
<td>F</td>
<td>0.38456783 10^{-3}</td>
<td>0.3845678300 10^{-3}</td>
<td>2.60033 10^{-11}</td>
</tr>
<tr>
<td>C (m^{-1})</td>
<td>0.18872246071 10^{-2}</td>
<td>0.1887224607 10^{-2}</td>
<td>5.29878 10^{-11}</td>
</tr>
<tr>
<td>D (m^{-1})</td>
<td>0.283038714728 10^{-2}</td>
<td>0.283038714727 10^{-2}</td>
<td>3.53317 10^{-11}</td>
</tr>
<tr>
<td>E (m^{-2})</td>
<td>0.31329007080 10^{-3}</td>
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The 5th order isotropic aberration coefficients without inclusion OMI.

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<thead>
<tr>
<th>Coefficient</th>
<th>DA Method</th>
<th>Aberration Integral</th>
<th>Relative error</th>
</tr>
</thead>
<tbody>
<tr>
<td>A_{5} (m)</td>
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<td>0.931253414197 10^{-2}</td>
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The 5th order isotropic aberration coefficients with inclusion OMI.

<table>
<thead>
<tr>
<th>Coefficient</th>
<th>DA Method</th>
<th>Aberration Integral</th>
<th>Relative error</th>
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<tbody>
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References

COSY INFINITY 10.
Kyoko Makino for important and valuable notes about
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Table 7
The 5th order anisotropic aberration coefficients without inclusion OMI.

<table>
<thead>
<tr>
<th>Coefficient</th>
<th>DA Method</th>
<th>Aberration Integral</th>
<th>Relative error</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a_5$ (m)</td>
<td>0.229990514369 $10^{-5}$</td>
<td>0.229990514369 $10^{-5}$</td>
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Table 8
The 5th order anisotropic aberration coefficients with inclusion OMI.

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<tr>
<th>Coefficient</th>
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effective, and accurate for high order aberration
analysis.

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References