

Third- and Fifth-Order Geometric Aberrations in Magnetic Exponential Lens Model for Object Magnetic Immersion

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Third- and Fifth-Order Geometric Aberrations in Magnetic Exponential Lens Model for Object Magnetic Immersion

Abstract

We numerically employ the differential algebraic technique to calculate the third and fifth-order geometric aberrations coefficients, which are derived by using the map method, of the magnetic exponential lens model. These coefficients are calculated for object magnetic immersion (OMI). The magnetic exponential model is used as an example for the magnetic round lens to calculate the coefficients. The numerical electron optical results are perfectly in match with the analytically results with a very small relative error (10^{-10} - 10^{-12}). The DA advantage technique is a very compendious, effective, proper, and accurate for analysis of electron optical aberration. The COSYINFINITY 10 code is used in our calculations.

Keywords

Differential algebra, Map method, Magnetic round lens, high order Geometric aberrations, OMI effect, COSYINFINITY 10

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Cover Page Footnote

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1. Introduction

Through the growing applications of high-quality systems of focusing charged particle beams such as high-resolution electron microscopes, electron and ion lithography systems, and beam accelerators, the desire to design practical electromagnetic lenses with aberrations of higher orders than third-order is receiving international attention [1–3]. Therefore, it is required to study the higher-order aberration for such focusing systems. Differential algebra (DA) is a highly accurate and advanced tool for solving sets of nonlinear dynamical equations as in the high-order aberrations of systems of electron-optical lenses. The DA method is a very accurate method with no special requirements for the calculation, developed by many authors including Berz [4], Wang and Chen [5,6] and Liu [7]; it can be applied to problems in the electric and/or magnetic fields engineering design. In this work, we utilize the DA method and COSY INFINITY 10 program to calculate practical electron lenses with magnetic fields that include object magnetic immersion (OMI) corrections. The OMI correction method can be used to correct chromatic and spherical aberration coefficients for magnetic lenses when the object is located in the field of the magnetic lens. The Gaussian properties and the third and fifth orders aberrations are determined for an exponential magnetic electron lens whose magnetic field magnitudes are input into a discretized array structure for further analysis. Three DA variable types exist for analyzing the aberration of rotationally symmetric electron optical systems: the fixed coordinates DA description, the rotating coordinates DA description, and the hybrid coordinates DA description [7,8]. The rotational coordinate system is used to facilitate the analytical paraxial ray equation. In the magnetic lens case, if the object immersed in the field, the correction of aberration done using OMI effect [9]. In the case of higher-order aberration (Fifth and Seventh), the description of DA in rotational coordinates becomes very complicated. We derive the general equation of electron trajectory and we obtain the transfer map of the rotational coordinates by tracking the trajectory equation. Seman [10] was the first researcher who discussed the problems of object magnetic immersion OMI in magnetic lenses. The high order aberrations (third and fifth) for the magnetic exponential model are carried out using the DA technique. In this work, the DA technique using COSY INFINITY 10 [11,12] is used for such calculations for the first time.

The DA results were crosschecked using the high order (third and fifth) aberration integrals evaluated by Wolfram Mathematica 9 program [13]. The results proved to be very compendious, effective, and accurate. The expression of the general equation of electron trajectory in rotating and fixed coordinates are given in many references [14–19]. It performs a significant part in clarifying the differential algebraic method in rotational coordinates.

2. DA description types of high order geometrical aberrations

Tracking the equation of the general trajectory of the electrons is used in fixed coordinates to obtain the higher orders (third and fifth) geometric aberrations using the differential algebra integrator limit starting at z_o (plane of object) to z_i (plane of image) in fixed coordinate [18];

$$\begin{aligned} \Delta X_{3i} &= \sum_{k,l,m,n=0,1,2,3}^{k+l+m+n=3} M_f(1.klmn) X_o^k X_o^l Y_o^m Y_o^n, \\ \Delta Y_{3i} &= \sum_{k,l,m,n=0,1,2,3}^{k+l+m+n=3} M_f(3.klmn) X_o^k X_o^l Y_o^m Y_o^n, \\ \Delta X_{5i} &= \sum_{k,l,m,n=0,1,2,3,4,5}^{k+l+m+n=5} M_f(1.klmn) X_o^k X_o^l Y_o^m Y_o^n, \\ \Delta Y_{5i} &= \sum_{k,l,m,n=0,1,2,3,4,5}^{k+l+m+n=5} M_f(3.klmn) X_o^k X_o^l Y_o^m Y_o^n \end{aligned} \quad (1)$$

where $M_f(i,klmn)$ is the transfer map elements in fixed coordinates, the number of orders depending on the sum of k , l , m , and n . While tracking the expression of rotating transform coordinates the DA characterization of rotational coordinates are calculated [16,18] as in the following form;

$$\begin{aligned} \Delta x_{3i} &= \sum_{k,l,m,n=0,1,2,3}^{k+l+m+n=3} M_r(1.klmn) x_o^k x_o^l y_o^m y_o^n, \\ \Delta y_{3i} &= \sum_{k,l,m,n=0,1,2,3}^{k+l+m+n=3} M_r(3.klmn) x_o^k x_o^l y_o^m y_o^n, \\ \Delta x_{5i} &= \sum_{k,l,m,n=0,1,\dots,5}^{k+l+m+n=5} M_r(1.klmn) x_o^k x_o^l y_o^m y_o^n, \\ \Delta y_{5i} &= \sum_{k,l,m,n=0,1,\dots,5}^{k+l+m+n=5} M_r(3.klmn) x_o^k x_o^l y_o^m y_o^n \end{aligned} \quad (2)$$

where M_r (i, klmn) is the transfer map elements in the rotating coordinates. The third-order geometrical aberrations have an expansion, in the vector form, of combined electromagnetic lenses

are [14]; $B = M_r(1.0300)$, $F = M_r(1.1002)$, $C = \frac{M_r(1.1011)}{2}$, $D = M_r(1.0120)$, $E = M_r(1.3000)$, $f = M_r(3.1002)/3$, $c = M_r(3.1011)/2$, $e = M_r(3.3000)$ and the fifth order coefficients are, $A_5 = M_r(1.0500)$, $B_{51} = M_r(1.1004)$, $B_{52} =$

$$\begin{aligned} \Delta \vec{r}_3 = & B \left[\vec{r}_o' \cdot \vec{r}_o' \right] \vec{r}_o' + F \left[\left(\vec{r}_o' \cdot \vec{r}_o' \right) \vec{r}_o + 2 \left(\vec{r}_o' \cdot \vec{r}_o \right) \vec{r}_o \right] + 2C \left[\vec{r}_o' \cdot \vec{r}_o \right] \vec{r}_o + D \left[\vec{r}_o \cdot \vec{r}_o \right] \vec{r}_o' \\ & + E \left[\vec{r}_o \cdot \vec{r}_o \right] \vec{r}_o + f \left[3 \left(\vec{r}_o' \cdot \vec{r}_o' \right) \vec{r}_o'^* - 2 \left(\vec{r}_o' \cdot \vec{r}_o \right) \vec{r}_o'^* \right] + c \left[2 \left(\vec{r}_o' \cdot \vec{r}_o \right) \vec{r}_o'^* - \left(\vec{r}_o \cdot \vec{r}_o \right) \vec{r}_o'^* \right] \\ & + e \left[\left(\vec{r}_o \cdot \vec{r}_o \right) \vec{r}_o'^* \right] \end{aligned} \quad (3)$$

$$\begin{aligned} \vec{r}_o = (x_o, y_o), \quad \vec{r}_o' = (x_o', y_o'), \quad \vec{r}_o^* \\ = (-y_o, x_o), \quad \vec{r}_o'^* = (-y_o', x_o') \end{aligned}$$

In addition, the expansion, for combined electromagnetic lenses, of the fifth order geometrical aberrations in a vector form is;

$M_r(1.0311)$, $C_{51} = M_r(1.1013)$, $C_{52} = M_r(1.0320)$.
 $C_{53} = M_r(1.0122) - C_{52}$, $D_{51} = M_r(1.3002)$, $D_{52} = M_r(1.1022) - D_{51}$, $D_{53} = M_r(1.0131)$, $E_{51} = M_r(1.3011)$, $E_{52} = M_r(1.0140)$, $F_5 = M_r(1.5000)$, $a_5 = M_r(3.0500)$, $b_{51} = M_r(3.1004)$, $b_{52} = M_r(3.0311)$, $c_{51} = M_r(3.1013)$, $c_{52} = M_r(3.0320)$, $c_{53} =$

$$\begin{aligned} \Delta \vec{r}_{5i} = & \left(\vec{r}_o' \cdot \vec{r}_o' \right)^2 \left(A_5 \vec{r}_o' + a_5 \vec{r}_o'^* \right) + \left(\vec{r}_o' \cdot \vec{r}_o \right)^2 \left(B_{51} \vec{r}_o + b_{51} \vec{r}_o^* \right) \\ & + \left(\vec{r}_o' \cdot \vec{r}_o \right) \left(\vec{r}_o' \cdot \vec{r}_o \right) \left(B_{52} \vec{r}_o' + b_{52} \vec{r}_o'^* \right) + \left(\vec{r}_o' \cdot \vec{r}_o \right) \left(\vec{r}_o \cdot \vec{r}_o \right) \left(C_{51} \vec{r}_o + c_{51} \vec{r}_o^* \right) \\ & + \left(\vec{r}_o' \cdot \vec{r}_o \right) \left(\vec{r}_o \cdot \vec{r}_o \right) \left(C_{52} \vec{r}_o' + c_{52} \vec{r}_o'^* \right) + \left(\vec{r}_o' \cdot \vec{r}_o \right)^2 \left(C_{53} \vec{r}_o' + c_{53} \vec{r}_o'^* \right) \\ & + \left(\vec{r}_o' \cdot \vec{r}_o \right) \left(\vec{r}_o \cdot \vec{r}_o \right) \left(D_{51} \vec{r}_o + d_{51} \vec{r}_o^* \right) + \left(\vec{r}_o' \cdot \vec{r}_o \right)^2 \left(D_{52} \vec{r}_o + d_{52} \vec{r}_o^* \right) + \\ & \left(\vec{r}_o' \cdot \vec{r}_o \right) \left(\vec{r}_o \cdot \vec{r}_o \right) \left(D_{53} \vec{r}_o' + d_{53} \vec{r}_o'^* \right) + \left(\vec{r}_o' \cdot \vec{r}_o \right) \left(\vec{r}_o \cdot \vec{r}_o \right) \left(E_{51} \vec{r}_o + e_{51} \vec{r}_o^* \right) \\ & + \left(\vec{r}_o \cdot \vec{r}_o \right)^2 \left(E_{52} \vec{r}_o' + e_{52} \vec{r}_o'^* \right) + \left(\vec{r}_o \cdot \vec{r}_o \right)^2 \left(F_5 \vec{r}_o + f_5 \vec{r}_o^* \right), \\ \vec{r}_o = (x_o, y_o), \quad \vec{r}_o' = (x_o', y_o'), \quad \vec{r}_o^* = (-y_o, x_o), \quad \vec{r}_o'^* = (-y_o', x_o'), \end{aligned} \quad (4)$$

In comparison equation (2) with equations (3) and (4), the connection between the elements of the map in Eq. (2) and the coefficients of aberration in equations (3) and (4) are obtained. The Glaser's notation of the third order coefficients of the geometrical aberration

$M_r(3.0122) - c_{52}$, $d_{51} = M_r(3.3002)$, $d_{52} = M_r(3.1022) - d_{51}$, $d_{53} = M_r(3.0131)$, $e_{51} = M_r(3.3011)$, $e_{52} = M_r(3.0140)$, $f_5 = M_r(3.5000)$.

The hybrid coordinates DA description have the form [3],

$$\begin{aligned} \Delta x_{3mi} &= \sum_{k.l.m.n=0.1.2.3}^{k+l+m+n=3} M_h(1.klmn)X_o^kX_o^lY_o^mY_o^n, \\ \Delta y_{3mi} &= \sum_{k.l.m.n=0.1.2.3}^{k+l+m+n=3} M_h(3.klmn)X_o^kX_o^lY_o^mY_o^n, \\ \Delta x_{5mi} &= \sum_{k.l.m.n=0.1.2.3.4.5}^{k+l+m+n=5} M_h(1.klmn)X_o^kX_o^lY_o^mY_o^n, \\ \Delta x_{5mi} &= \sum_{k.l.m.n=0.1.2.3.4.5}^{k+l+m+n=5} M_h(3.klmn)X_o^kX_o^lY_o^mY_o^n \end{aligned} \tag{5}$$

where the suffix "m" indicates using the effect of OMI, the aberrations were corrected. The hybrid coordinates transfer map elements are represented by $M_h(1.klmn)$ and $M_h(3.klmn)$, where they could be derived from $M_f(1.klmn)$ and $M_f(3.klmn)$. The relation between the geometrical aberration coefficients and the hybrid coordinates map elements together with the containment of the effect of OMI is calculated. The aberration coefficients of the third order isotropic are:

$$\begin{aligned} B_m &= M_h(1.0300) = M_f(3.0300)\sin \theta_i + M_f(1.0300)\cos \theta_i \\ F_m &= M_h(1.1002) = M_f(3.1002)\sin \theta_i + M_f(1.1002)\cos \theta_i \\ C_m &= M_h(1.1011) = [M_f(3.1011)\sin \theta_i + M_f(1.1011)\cos \theta_i] / 2 \\ D_m &= M_h(1.0120) = M_f(3.0120)\sin \theta_i + M_f(1.0120)\cos \theta_i \\ E_m &= M_h(1.3000) = M_f(3.3000)\sin \theta_i + M_f(1.3000)\cos \theta_i \end{aligned} \tag{6}$$

The aberration coefficients of the third order anisotropic are:

$$\begin{aligned} f_m &= M_h(3.1002) = [-M_f(1.1002)\sin \theta_i + M_f(3.1002)\cos \theta_i] / 3 \\ c_m &= M_h(3.1011) = [-M_f(1.1011)\sin \theta_i + M_f(3.1011)\cos \theta_i] / 2 \\ e_m &= M_h(3.3000) = [-M_f(1.3000)\sin \theta_i + M_f(3.3000)\cos \theta_i] \end{aligned} \tag{7}$$

The aberration coefficients of the fifth order isotropic are:

$$\begin{aligned} A_{5m} &= M_h(1, 0500) = M_f(3, 0500)\sin \theta_i + M_f(1, 0500)\cos \theta_i \\ B_{51m} &= M_h(1, 1004) = M_f(3, 1004)\sin \theta_i + M_f(1, 1004)\cos \theta_i \\ B_{52m} &= M_h(1, 0311) = M_f(3, 0311)\sin \theta_i + M_f(1, 0311)\cos \theta_i \\ C_{51m} &= M_h(1, 1013) = M_f(3, 1013)\sin \theta_i + M_f(1, 1013)\cos \theta_i \\ C_{52m} &= M_h(1, 0320) = M_f(3, 0320)\sin \theta_i + M_f(1, 0320)\cos \theta_i \end{aligned} \tag{8}$$

$$\begin{aligned} C_{53m} &= M_h(1, 0122) - C_{52m} = M_f(3, 0122)\sin \theta_i + M_f(1, 0122)\cos \theta_i - C_{52m} \\ D_{51m} &= M_h(1, 3002) = M_f(3, 3002)\sin \theta_i + M_f(1, 3002)\cos \theta_i \\ D_{52m} &= M_h(1, 1022) - D_{51m} = M_f(3, 1022)\sin \theta_i + M_f(1, 1022)\cos \theta_i - D_{51m} \\ D_{53m} &= M_h(1, 0131) = M_f(3, 0131)\sin \theta_i + M_f(1, 0131)\cos \theta_i \\ E_{51m} &= M_h(1, 3011) = M_f(3, 3011)\sin \theta_i + M_f(1, 3011)\cos \theta_i \\ E_{52m} &= M_h(1, 0140) = M_f(3, 0140)\sin \theta_i + M_f(1, 0140)\cos \theta_i \\ F_{5m} &= M_h(1, 5000) = M_f(3, 5000)\sin \theta_i + M_f(1, 5000)\cos \theta_i \end{aligned} \tag{9}$$

The aberration coefficients of the fifth order anisotropic are:

$$\begin{aligned} a_{5m} &= M_h(3, 0500) = -M_f(1, 0500)\sin \theta_i + M_f(3, 0500)\cos \theta_i \\ b_{52m} &= M_h(3, 0311) = -M_f(1, 0311)\sin \theta_i + M_f(3, 0311)\cos \theta_i \\ c_{51m} &= M_h(3, 1013) = -M_f(1, 1013)\sin \theta_i + M_f(3, 1013)\cos \theta_i \\ c_{52m} &= M_h(3, 0320) = -M_f(1, 0320)\sin \theta_i + M_f(3, 0320)\cos \theta_i \end{aligned} \tag{10}$$

$$\begin{aligned} c_{53m} &= M_h(3, 0122) - c_{52m} = -M_f(1, 0122)\sin \theta_i + M_f(3, 0122)\cos \theta_i - c_{52m} \\ d_{51m} &= M_h(3.3002) = -M_f(1.3002)\sin \vartheta_i + M_f(3.0300)\cos \vartheta_i \\ d_{52m} &= M_h(3.1022) - d_{51m} = -M_f(1.1022)\sin \vartheta_i + M_f(3.1022)\cos \vartheta_i - d_{51m} \\ d_{53m} &= M_h(3.0131) = -M_f(1.0131)\sin \vartheta_i + M_f(3.0131)\cos \vartheta_i \\ e_{51m} &= M_h(3.3011) = -M_f(1.3011)\sin \vartheta_i + M_f(3.3011)\cos \vartheta_i \\ e_{52m} &= M_h(3.0140) = -M_f(1.0140)\sin \vartheta_i + M_f(3.0140)\cos \vartheta_i \\ f_{5m} &= M_h(3.5000) = -M_f(1.5000)\sin \vartheta_i + M_f(3.5000)\cos \vartheta_i \end{aligned} \tag{11}$$

3. High order aberrations of the magnetic lens

The concept of higher-order aberration coefficients derivation is the same as that in Refs. [18,19]. According to equation (3), there are three anisotropic (lowercase letters) and five isotropic (uppercase letters) coefficients of the third-order aberration: the symbol B refers to the spherical aberration, the symbols F and *f* refers to coma, the symbols C and *c* refers to astigmatism, the symbol D refers to the field curvature, and the symbols E and *e*

the symbols *e*₅₂ and *E*₅₂ for field curvature, and finally the symbols *f*₅ and *F*₅ refer to the distortion [20–22]. The high order coefficients formulas for OMI correction are given as follows;

$$\begin{aligned} B_m &= B, \quad F_m = F, & C_m &= C + 2\theta'_o f + \theta'^2_o B, \\ D_m &= D - 6\theta'_o f + 3\theta'^2_o B, & E_m &= E - 2\theta'_o c + \theta'^2_o F. \\ f_m &= f - \theta'_o B, \quad c_m = c - \theta'_o F. & e_m &= e - \theta'_o D + 3\theta'^2_o f - \theta'^3_o B \end{aligned} \tag{12}$$

$$\begin{aligned} A_{5m} &= A_5, \quad B_{51m} = B_{51} + 5\theta'_o a_5, \\ B_{52m} &= B_{52} - 4\theta'_o a_5, \\ C_{51m} &= C_{51} + \theta'_o (4b_{51} + 3b_{52}) - 4\theta'^2_o A_5, \\ C_{52m} &= C_{52} - 4\theta'_o b_{51} + 10\theta'^2_o A_5, \\ C_{53m} &= C_{53} - 2\theta'_o b_{52} - 4\theta'^2_o A_5, \\ D_{51m} &= D_{51} + 3\theta'_o c_{52} + 6\theta'^2_o B_{51} + 10\theta'^3_o a_5, \\ D_{52m} &= D_{52} + \theta'_o (2c_{51} + c_{53}) - 2\theta'^2_o (2B_{51} + B_{52}) - 4\theta'^3_o a_5, \\ D_{53m} &= D_{53} - 2\theta'_o (c_{51} + c_{53}) - 3\theta'^2_o B_{52} - 4\theta'^3_o a_5, \\ E_{51m} &= E_{51} + \theta'_o (2d_{51} + d_{53}) + \theta'^2_o (C_{51} - 2C_{52}) + \theta'^3_o (4b_{51} + b_{52}) - 4\theta'^4_o A_5, \\ E_{52m} &= E_{52} - 2\theta'_o d_{51} + 3\theta'^2_o C_{52} - 4\theta'^3_o b_{51} + 5\theta'^4_o A_5, \\ F_{5m} &= F_5 + \theta'_o e_{52} + \theta'^2_o D_{51} + \theta'^3_o c_{52} + \theta'^4_o B_{51} + \theta'^5_o a_5; \end{aligned} \tag{13}$$

$$\begin{aligned} a_{5m} &= a_5, \quad b_{51m} = b_{51} - 5\theta'_o A_5, \quad b_{52m} = b_{52} + 4\theta'_o A_5, \\ c_{51m} &= c_{51} - \theta'_o (4B_{51} + 3B_{52}) - 4\theta'^2_o a_5, \\ c_{52m} &= c_{52} + 4\theta'_o B_{51} + 10\theta'^2_o a_5, \\ c_{53m} &= c_{53} + 2\theta'_o B_{52} - 4\theta'^2_o a_5, \\ d_{51m} &= d_{51} - 3\theta'_o C_{52} + 6\theta'^2_o b_{51} - 10\theta'^3_o A_5, \\ d_{52m} &= d_{52} - \theta'_o (2C_{51} + C_{53}) - 2\theta'^2_o (2b_{51} + b_{52}) + 4\theta'^3_o A_5, \\ d_{53m} &= d_{53} + 2\theta'_o (C_{51} + C_{52}) + 3\theta'^2_o b_{52} + 4\theta'^3_o A_5, \\ e_{51m} &= e_{51} - \theta'_o (2D_{51} + D_{53}) + \theta'^2_o (c_{51} - 2c_{52}) + \theta'^3_o (4B_{51} + B_{52}) - 4\theta'^4_o a_5, \\ e_{52m} &= e_{52} + 2\theta'_o D_{51} + 3\theta'^2_o c_{52} + 4\theta'^3_o B_{51} + 5\theta'^4_o a_5, \\ f_{5m} &= f_5 - \theta'_o E_{52} + \theta'^2_o d_{51} - \theta'^3_o C_{52} + \theta'^4_o b_{51} - \theta'^5_o A_5. \end{aligned} \tag{14}$$

refers to the distortion. According to Equation (4), there are twelve isotropic and twelve anisotropic fifth-order coefficients, where the uppercase symbols are for isotropic and the lowercase symbols are for anisotropic coefficients. For spherical aberration we use the symbols *a*₅ and *A*₅, the symbols *b*₅₁, *b*₅₂, and *B*₅₁, *B*₅₂ for coma, the symbols *c*₅₁, *c*₅₂, *c*₅₃ and *C*₅₁, *C*₅₂, *C*₅₃ for the peanut aberration, the symbols *d*₅₁, *d*₅₂, *d*₅₃ and *D*₅₁, *D*₅₂, *D*₅₃ for the elliptical coma, the symbols *e*₅₁ and *E*₅₁ for astigmatism,

where $\theta'_o = \frac{\eta}{2} \frac{B(z_o)}{\sqrt{V(z_o)}}$, $\eta = \sqrt{\frac{e}{2m}}$, *B*(*z*_o) and *V*(*z*_o) are the magnetic induction and potential function respectively at the object plane *z*_o.

4. Computational illustration

The former derivations were a concise description of the basics of the high order geometric aberrations calculation for electron lenses with magnetic fields that

include OMI corrections. More information about the DA method can be found in Berz (1999) [22]. The formulation of the DA method is independent of the aberration order, thus, to understand the form of the Hamiltonian an analytical description of the field must be defined. Thus, we must know the exact form of the axial potentials and their derivatives, to determine any order of aberrations. Therefore, the inaccuracy of the axial potential higher-order derivatives is a limit for the use of the DA method in real optical systems, i.e. the analytical field model (or the axial potential) must have a trajectory equation that can be solved analytically and its higher order aberrations can also be expressed by the analytical expression of the field model to have an accurate solution [23,24].

In the present work, the exponential model of the magnetic round lens is one of the well-known models that can make the paraxial ray equation soluble and useful, especially for ‘single-pole’ lenses [25], whose magnetic induction distributions have the following form

$$B(z) = B_0 e^{-\left[\frac{z}{d}\right]^2} \tag{15}$$

where $B_0 = 0.01$ T is the maximum magnetic flux and d is the half-width of the magnetic flux distribution in mm. The axial magnetic flux distributions are $B(z)$ shown in Fig. 1, along z (optical axis) from $z_o = -0.3$ mm (object plane) to $z_i = 0.3$ mm (image plane) at different values of d ($d = 0.1, 0.09, 0.08, 0.07, 0.06, 0.05$ mm). Two packages are used: the COSY INFINITY10 [11,12,26] and Wolfram Mathematica version 9 [13]. With the aid of the 8th order Runge-Kutta integrator [27], we calculate the transfer map produced by tracking the trajectory equations in rotating and fixed coordinates. All the high order aberration coefficients (third and fifth) are calculated

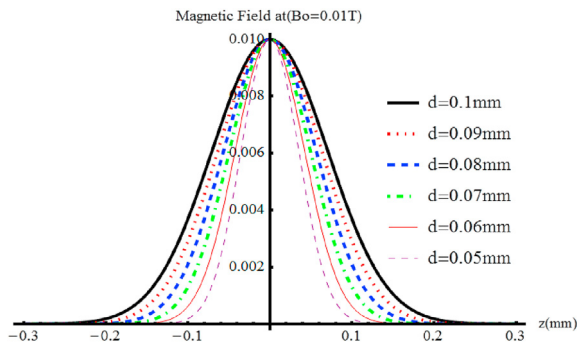


Fig. 1. The axial magnetic fields $B(z)$ as a function of different values of (d) along the optical axis (z) with $B_0 = 0.01$ T.

using two packages (without and with the OMI participation) for the magnetic exponential model. The spherical aberration disc Δr (disc of least confusion) is also calculated using the following equation [28,29]:

$$\Delta r = MB\alpha_o^3. \tag{16}$$

where M is the magnification, B (in some reference denoted by C_s) is the third order spherical aberration coefficient and α_o is the half angle in radian and we take a range of $(\alpha_o)^3$ from zero to 0.2 radian.

5. Discussion and conclusion

The third- and fifth-order isotropic and anisotropic geometric aberration with and without the OMI is calculated for the magnetic exponential lens model. The equation of the paraxial trajectory in fixed coordinates is directly derived from the equation of general trajectory. However, in the DA technique, the trajectory equation in rotating coordinates becomes most important because it makes the DA description very easy and straight forward to expand to higher-order aberrations. Table 1 represents the results of the magnetic exponential model lens. The optical properties are calculated by two methods: the DA and the analytical methods under the two parameters (1) $d = 0.1$ mm, and (2) $B_0 = 0.01$ T. One can notice from this table that the results are very precise with a very small relative error of order (10^{-9} to 10^{-11}) and agreement between the two methods are very good. Table 2 shows the coefficients of third-order aberration as a function of the half-width (d) for the magnetic exponential model lens. The variation of the geometric aberration coefficients with d is shown in Fig. 2. From the table and the figure, one can see that astigmatism, field curvature, and the distortion (symbols C, D, and E respectively) are decreasing with increasing d under a constant value of B_0 (0.1 T). This is because of the nature of these aberrations and the effect of the magnetic field distribution as shown in Fig. 1. However, the Coma (symbol F) is constant and the spherical

Table 1

Optical properties for magnetic exponential model lens calculated by DA and analytical methods under the conditions $d = 0.1$ mm and $B_0 = 0.01$ T.

	$-1/f_i$	M	M_s
Analytic method	-3.176108987312	1	1
DA method	-3.176108987274	0.999999991455	1.000000000854
Relative error	-3.8×10^{-11}	8.545×10^{-9}	-8.54×10^{-9}

Table 2
Coefficients of the 3rd order aberration for various half-width (d) under $B_o = 0.01$ T.

$d(m) \times 10^{-3}$	B (m)	F	C (m^{-1})	D (m^{-1})	E (m^{-2})
0.1	0.268931×10^{-2}	0.384567×10^{-3}	0.188722×10^{-2}	0.283038×10^{-2}	0.313290×10^{-3}
0.09	0.242038×10^{-2}	0.384567×10^{-3}	0.209691×10^{-2}	0.314487×10^{-2}	0.386775×10^{-3}
0.08	0.215145×10^{-2}	0.384566×10^{-3}	0.235903×10^{-2}	0.353798×10^{-2}	0.489510×10^{-3}
0.07	0.188251×10^{-2}	0.384563×10^{-3}	0.269603×10^{-2}	0.404341×10^{-2}	0.639360×10^{-3}
0.06	0.161358×10^{-2}	0.384561×10^{-3}	0.314537×10^{-2}	0.471731×10^{-2}	0.870239×10^{-3}
0.05	0.134465×10^{-2}	0.384558×10^{-3}	0.277444×10^{-2}	0.566077×10^{-2}	0.125315×10^{-3}

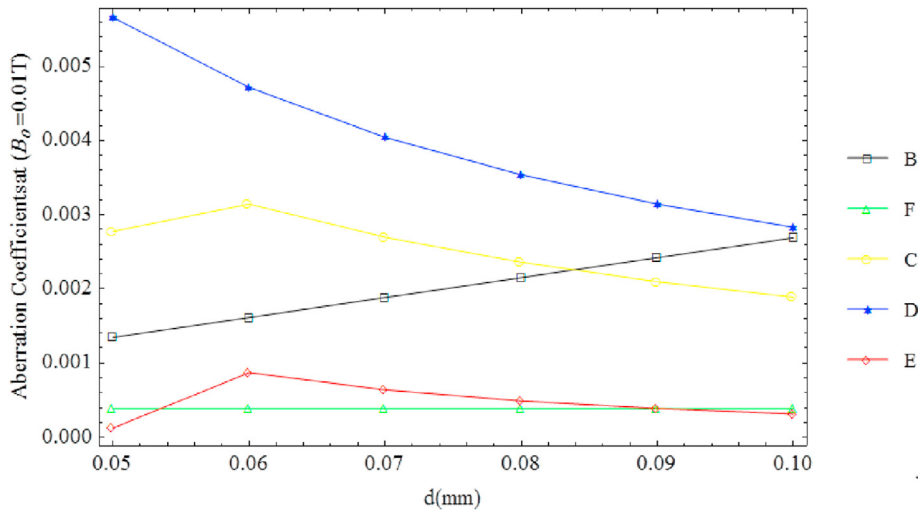


Fig. 2. Various geometrical aberration coefficients as a function of different values of half-width (d) under the maximum field $B_o = 0.01$ T.

Table 3
The 3rd order aberration coefficients for various maximum field B_o under $d = 0.1$ mm.

B_o (T)	B (m)	F	C (m^{-1})	D (m^{-1})	E (m^{-2})
0.01	0.268931×10^{-2}	0.384566×10^{-3}	0.188722×10^{-2}	0.283038×10^{-2}	0.313290×10^{-3}
0.05	0.674845×10^{-2}	0.240756×10^{-3}	0.473470×10^{-2}	0.707430×10^{-1}	0.195159×10^{-2}
0.1	0.273104×10^{-1}	0.387239×10^{-2}	0.191355×10^{-1}	0.282775	0.309090×10^{-1}
0.15	0.626512×10^{-1}	0.197778×10^{-1}	0.437067×10^{-1}	0.635592	0.153899
0.2	0.114432	0.632957×10^{-1}	0.789980×10^{-1}	1.12864	0.475585
0.25	0.385096	0.257083	0.125153	1.26178	1.12951

aberration coefficient (symbol B) is decreasing with decreasing d, this is because of the spherical aberration dependent on the half-width of the magnetic field distribution which is decreasing with d as shown in Fig. 1.

Table 3 and Fig. 3 show the results of the DA for the coefficients of third-order aberrations under various maximum field B_o for the magnetic exponential lens model under $d = 0.1$ mm. It is obvious that the geometrical coefficients of third-order aberration increase as the values of B_o rises. Plus, the distortion and

field curvature aberrations coefficients are more affected with the increasing of B_o . Fig. 4 shows the relationship between spherical aberration disc Δr , calculated using equation (16), and the $(\alpha o)^3$ under fixed magnification value $M = 0.9999$ for magnetic exponential lens model. This figure shows the disc of spherical aberration has acceptable values for $(\alpha o)^3$ values up to less than 0.2 rad.

The results of geometrical aberration coefficients of third-order (isotropic and anisotropic) using the DA method for the magnetic exponential model with and

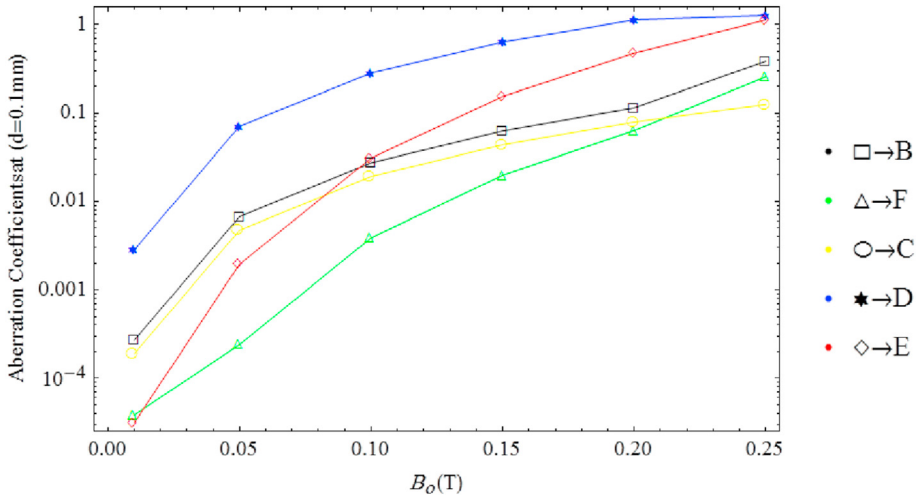


Fig. 3. Various geometrical aberration coefficients under different values of B_o for $d = 0.1$ mm.

without the OMI effect for the exact conditions used in Table 1 are shown in Table 4. The DA and the analytical methods result are shown in Tables 5–8, for the coefficients fifth-order geometrical aberration (isotropic and anisotropic) for the magnetic lens exponential model with and without OMI effect for the exact conditions in Table 1 respectively. Table 4, proves that the values of third-order spherical aberration and coma are equal as listed in equation (6). For the fifth-order spherical coefficients for isotropic and anisotropic are equal as in Tables 5 and 6 according to equations (7) and (8), respectively. The other

coefficients which are listed in Tables 4–8 are computed with high precision by two methods for cross-checking. The first method is the DA method calculated using the COSYINFINITY 10 and the second method is the aberration integral method [16,25] computed using the Mathematica 9 program. It is confident that the two methods are in good agreement with a very small error of order (10^{-10} - 10^{-12}). The present research proved that the technique using the DA method with COSYINFINITY 10 is an excellent tool for such calculations, and it is very compendious,

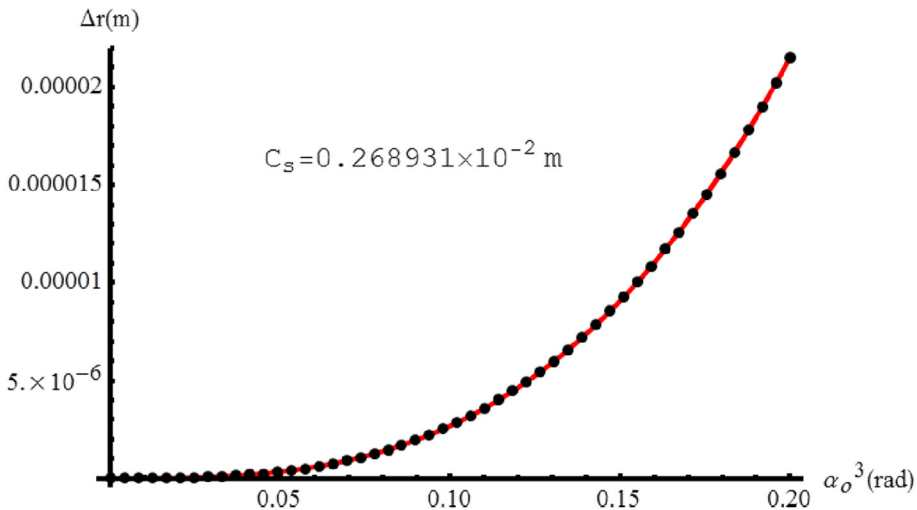


Fig. 4. Represent the relation between $(\alpha_o)^3$ and the disc Δr under the magnification $M = 0.9999$ for a magnetic exponential model lens.

Table 4
The 3rd order coefficients of aberration with and without inclusion OMI effect.

Coefficient	DA Method	Aberration Integral	Relative error
B (m)	0.268931339128 10^{-2}	0.268931339126 10^{-2}	7.43687 10^{-12}
F	0.38456783501 10^{-3}	0.38456783500 10^{-3}	2.60033 10^{-11}
C (m^{-1})	0.18872246071 10^{-2}	0.18872246070 10^{-2}	5.29878 10^{-11}
D (m^{-1})	0.283038717428 10^{-2}	0.283038717427 10^{-2}	3.53317 10^{-12}
E (m^{-2})	0.31329007080 10^{-3}	0.31329007078 10^{-3}	6.38386 10^{-11}
F	0.637803310005 10^{-5}	0.637803310002 10^{-5}	4.70368 10^{-12}
c (m^{-1})	0.198149596904 10^{-1}	0.198149596901 10^{-1}	1.51401 10^{-11}
e (m^{-2})	0.249950926993 10^{-2}	0.249950926990 10^{-2}	1.20025 10^{-11}
B_m (m)	0.268931339128 10^{-2}	0.268931339126 10^{-2}	7.43687 10^{-12}
F_m	0.38456783501 10^{-3}	0.38456783500 10^{-3}	2.60033 10^{-11}
C_m (m^{-1})	0.18872134623 10^{-2}	0.18872134621 10^{-2}	1.05976 10^{-10}
D_m (m^{-1})	0.283038234126 10^{-2}	0.283038234122 10^{-2}	1.41323 10^{-11}
E_m (m^{-2})	0.31328865412 10^{-3}	0.31328865409 10^{-3}	9.57585 10^{-11}
f_m	0.117852587460 10^{-4}	0.117852587458 10^{-4}	1.69705 10^{-11}
c_m (m^{-1})	0.198136696323 10^{-1}	0.198136696321 10^{-1}	1.00939 10^{-11}
e_m (m^{-2})	0.194594999224 10^{-2}	0.194594999220 10^{-2}	2.05555 10^{-11}

Table 5
The 5th order isotropic aberration coefficients without inclusion OMI.

Coefficient	DA Method	Aberration Integral	Relative error
A_5 (m)	0.931253414197 10^{-2}	0.931253414197 10^{-2}	0
B_{51}	0.288216273304 10^{-2}	0.288216273301 10^{-2}	1.0409 10^{-11}
B_{52}	0.182823992662 10^{-2}	0.182823992660 10^{-2}	1.09395 10^{-11}
C_{51} (m^{-1})	0.141807872152	0.141807872149	2.11553 10^{-11}
C_{52} (m^{-1})	0.118681906042 10^1	0.118681906042 10^1	0
C_{53} (m^{-1})	0.141287570572	0.141287570570	1.41556 10^{-11}
D_{51} (m^{-2})	0.442053911493	0.442053911492	2.26212 10^{-12}
D_{52} (m^{-2})	0.493036611338 10^{-1}	0.493036611338 10^{-1}	0
D_{53} (m^{-2})	0.155757572463	0.155757572463	0
E_{51} (m^{-3})	0.296971216887 10^1	0.296971216884 10^1	1.0102 10^{-11}
E_{52} (m^{-3})	0.398051995876 10^2	0.398051995873 10^2	7.53666 10^{-12}
F_5 (m^{-4})	0.156360654477 10^2	0.156360654475 10^2	1.27909 10^{-11}

Table 6
The 5th order isotropic aberration coefficients with inclusion OMI.

Coefficient	DA Method	Aberration Integral	Relative error
A_{5m} (m)	0.931253414197 10^{-2}	0.931253414197 10^{-2}	0
B_{51m}	0.288214942464 10^{-2}	0.288214942461 10^{-2}	1.04089 10^{-11}
B_{52m}	0.182825057334 10^{-2}	0.182825057332 10^{-2}	1.09395 10^{-11}
C_{51m} (m^{-1})	0.141248593697	0.141248593696	7.07956 10^{-12}
C_{52m} (m^{-1})	0.118681963757 10^1	0.118681963755 10^1	1.68518 10^{-11}
C_{53m} (m^{-1})	0.141287952144	0.141287952141	2.12332 10^{-11}
D_{51m} (m^{-2})	0.441881332967	0.441881332967	0
D_{52m} (m^{-2})	0.496483690697 10^{-1}	0.496483690694 10^{-1}	6.04242 10^{-12}
D_{53m} (m^{-2})	0.155527900500	0.155527900495	3.21486 10^{-11}
E_{51m} (m^{-3})	0.296966329314 10^1	0.296966329311 10^1	1.01022 10^{-11}
E_{52m} (m^{-3})	0.398051939731 10^2	0.398051939727 10^2	1.00489 10^{-11}
F_{5m} (m^{-4})	0.156302530909 10^2	0.156302530906 10^2	1.91935 10^{-11}

Table 7
The 5th order anisotropic aberration coefficients without inclusion OMI.

Coefficient	DA Method	Aberration Integral	Relative error
a_5 (m)	0.229990514369 10^{-5}	0.229990514369 10^{-5}	0
b_{51}	0.979213080534 10^{-4}	0.979213080531 10^{-4}	3.06381 10^{-12}
b_{52}	0.186258538323 10^{-3}	0.186258538320 10^{-3}	1.61066 10^{-11}
c_{51} (m^{-1})	0.198652606142	0.198652606140	1.00679 10^{-11}
c_{52} (m^{-1})	0.497138529482 10^{-1}	0.497138529480 10^{-1}	4.02302 10^{-12}
c_{53} (m^{-1})	0.994282614349 10^{-1}	0.994282614346 10^{-1}	3.01735 10^{-12}
d_{51} (m^{-2})	0.447156881002 10^{-2}	0.447156881000 10^{-2}	4.47281 10^{-12}
d_{52} (m^{-2})	0.304936612769 10^{-1}	0.304936612768 10^{-1}	3.27948 10^{-12}
d_{53} (m^{-2})	0.486108121043 10^{-1}	0.486108121042 10^{-1}	2.05708 10^{-12}
e_{51} (m^{-3})	0.100331064244 10^2	0.100331064241 10^2	2.9901 10^{-11}
e_{52} (m^{-3})	0.502285030465 10^1	0.502285030463 10^1	3.9818 10^{-12}
f_5 (m^{-4})	0.997977018525	0.997977018524	1.002 10^{-12}

Table 8
The 5th order anisotropic aberration coefficients with inclusion OMI.

Coefficient	DA Method	Aberration Integral	Relative error
a_{5m} (m)	0.229990514369 10^{-5}	0.229990514369 10^{-5}	0
b_{51m}	0.151808286866 10^{-3}	0.151808286864 10^{-3}	1.31746 10^{-11}
b_{52m}	0.143148955273 10^{-3}	0.143148955271 10^{-3}	1.39715 10^{-11}
c_{51m} (m^{-1})	0.204123983826	0.204123983825	4.89901 10^{-12}
c_{52m} (m^{-1})	0.497005108710 10^{-1}	0.497005108707 10^{-1}	6.03608 10^{-12}
c_{53m} (m^{-1})	0.994240297786 10^{-1}	0.994240297785 10^{-1}	1.00569 10^{-12}
d_{51m} (m^{-2})	0.351050788405 10^{-3}	0.351050788402 10^{-3}	8.54574 10^{-12}
d_{52m} (m^{-2})	0.300019222401 10^{-1}	0.300019222398 10^{-1}	9.99935 10^{-12}
d_{53m} (m^{-2})	0.479555600780 10^{-1}	0.479555600777 10^{-1}	6.25586 10^{-12}
e_{51m} (m^{-3})	0.100319033847 10^2	0.100319033845 10^2	1.99364 10^{-11}
e_{52m} (m^{-3})	0.502182732501 10^1	0.502182732500 10^1	1.99113 10^{-12}
f_{5m} (m^{-4})	0.951910465150	0.951910465147	3.15164 10^{-12}

effective, and accurate for high order aberration analysis.

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