Multi-Step-Ahead Exchange Rate Forecasting For South Asian Countries Using Multi-Verse Optimized Multiplicative Functional Link Neural Networks

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Abstract
The dynamic nonlinearity approach, coupled with the exchange rate data series, makes its future predictions difficult. Sophisticated methods are highly desired for effective prediction of such data. Artificial neural networks (ANNs) have shown their ability to model and predict such data. This article presents a multi-verse optimizer (MVO) based multiplicative functional link neural network (MV-MFLN) model to forecast the exchange rate data. Functional link neural network (FLN) makes use of functional expansion for input data with a fewer number of adjustable neuron weights, which makes it capable of learning the uncertainties accompanying the exchange rate data. In contrast to the summation unit at the output layer of FLN, the proposed model uses a multiplicative unit to enhance the ability to learn the complex correlations within the input data. The MVO is employed to fine-tune the parameters of the MFLN. We validate the MV-MFLN on multi-step-ahead forecasting of six exchange rate series through the mean absolute percentage of error (MAPE) metrics. A comparative study with additional forecasts such as genetic algorithm based MFLN (GA-MFLN), differential evolution based MFLN (DE-MFLN), teaching-learning based optimization trained MFLN (TLB-MFLN), and gradient descent based MFLN (GD-MFLN) developed similarly is carried out. It is found that the proposed forecast produces the lowest MAPE values and quite capable of capturing the uncertainties associated with exchange rate data. Observations from comparative performance analysis suggest the superiority of the MV-MFLN-based forecast.

Keywords
exchange rate forecasting, financial time series data, multiplicative functional link neural network, multi-verse optimizer, genetic algorithm.

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1. Introduction

Forecasting exchange rate series is an exciting and vital area of interest for financial managers, economic traders, international companies, and all sorts of business stakeholders. However, the nonlinear behavior of exchange rate data makes it very difficult to predict.

Intelligent computational methods like ANN, fuzzy-based inference systems, and evolutionary computations have shown their efficiency in capturing the inherent nonlinearities of stock indices [1, 2], macroeconomic variables [3], and exchange rate time series [4–7]. ANN is a data-centric model that can approximate nonlinear patterns like financial time series data without any prior knowledge about the problem domain. Without any knowledge about the distribution of data patterns, it can learn from existing data and handle the inconsistency and messy information [8]. Usually, economic data are in numeric form. Hence, it is suitable to process such data by ANN without losing any information. The study conducted in Ref. [9] compared the forecast accuracy of the regression methods with ANN. Results indicate the superior performance of ANN than the regression methods. A case study [10] was conducted for the application of ANN to predict the future values of exchange rates. It showed superior performance of ANN based methods than its counterparts. Exchange rate datasets have high irregularity, fluctuations, and are liable to numerous errors [11]. Past decades have seen numerous applications of exciting statistical and linear models for exchange rate prediction [12–15]. However, these methods were based on the assumption that the exchange rate series are linear in nature and thus provide poor forecasting accuracy [16, 17].

Though multilayer ANNs are efficient in modeling the nonlinear data, issues like computational complexity and slower convergence rate are associated with them. The structural complexity of the network adds to the computational overhead and gives an ideal black-box concept. Alternatively, ANNs with higher-order components have shown stronger approximation capability, faster learning, and greater fault tolerance [18]. Only one set of trainable weights makes the network free from black-box visualization [19]. Higher-order components in the network can enhance the data storage capability and solve complex nonlinear tasks with a simple network having promising convergence capability. These ANNs belong to a special category called higher-order neural network (HONN). FLN is a popular and widely used HONN. The basic consideration of HONN is its single layered architecture. These models are computationally efficient due to the single-layered trainable weights. Several nonlinear basis functions are used in HONNs for functional expansion of inputs. Few forecasting application of financial time series data using HONNs are found in the literature [20–26]. These articles use variants of HONNs as well as their hybridizations with different nature-inspired learning techniques. Another benefit of FLN is that it reduces the concern of determining the network structure [27–30]. The hybridization of existing methodologies to achieve improved model performance is a common practice in the current era. Hybridization of ANN and artificial cooperative search algorithm for short-term electricity price forecasting provided enhanced accuracy compared to other methods [31]. The backtracking search algorithm was hybridized with an adaptive neuro-fuzzy inference system (ANFIS) [32] to predict the electricity price and showed promising results than ANN methods. The same method is applied to predict metallic conductor voltage, and the results obtained are superior to ANN, support vector regression, and ANFIS optimized by other techniques [33]. The artificial cooperative search algorithm was used for the long-term prediction of electricity consumption [34]. The method claimed that it is better than other well known optimization algorithms. A classical method for feedback linearization of a multivariable invertible nonlinear system through dynamic extension and state feedback was proposed in [35].

The efficiency of ANN-based forecasting techniques is mainly dependent on the training algorithm adopted for determining its tunable weights. The most commonly used ANN training method is the gradient-based backpropagation algorithm. However, the backpropagation algorithm has several drawbacks like slower convergence rate, trapping to local optima, inaccurate learning, and many others. As a result, the model takes quite a long time to meet the desired optima, thereby adds more computation overhead [36]. To overcome the above-said limitations, we have used a stochastic learning technique, namely MVO, for determining the optimal MFLN structure [37]. The MVO is a population-based optimization technique that can locate the global optima in the search space without compromising the learning rate. It maintains a
tradeoff between exploration and exploitation of the vested solution space by using the holes (white and black). Interested readers are suggested to read the article in Ref. [37], for the basic MVO method.

This article focuses on designing a structurally simple, fast and accurate forecasting model for exchange rate data. We have used MFLN as the base model. It has a single-layered architecture and thus is computationally efficient. Contrasting to the summation unit at the output neuron of traditional FLN, we have used the product unit at the output neuron [38]. Thus, the higher-order components generated through the functional expansion of inputs help in improving the learning ability of FLN. The MVO algorithm is used to fine-tune the weights of MFLN. The validation of the MV-MFLN model is done by using six exchange rate datasets. The obtained results are compared with other forecasting methods such as GA-MFLN, DE-MFLN, TLB-MFLN, and GD-MFLN. The major contributions of this paper are highlighted as follows:

- Use of a multiplicative unit instead of a summation unit at the output layer of FLN to enhance the learning ability of MFLN.
- Utilizing the search capability of MVO to fine-tune the MFLN parameters.

The remaining of this paper is structured as follows. The MVO, proposed MV-MFLN, and exchange rate datasets are discussed in Section 2. Interpretation of experimental results and discussions are carried out in Section 3. Conclusions are drawn and future works are suggested in Section 4.

2. Materials and methods

Here, we have described the methods and materials that form the core part of this paper. It includes the MVO algorithm and the proposed MV-MFLN-based forecasting method.

2.1. MVO

The multi-verse optimizer technique is a newly developed optimization method motivated by the concept of survival of numerous other universes along with holes (black, white, and worm types) that enable communication between each other [37]. The main characteristics of MVO are its population-based nature and guaranty to reach the global optima. The concept of black and white holes assists in the exploration of the search space, while the wormhole concept assists in the exploitation of the search space. The search process follows two simple steps. The first step includes interaction between two potential universes by exchanging objects (an object represents a variable of a solution). Secondly, the fitness of the universes are evaluated after assigning an inflation rate. The inflation rate decides the type of holes used to exchange objects within the universes. The higher and lower inflation rate values imply the kind of holes, i.e., white (sending objects) and black (receiving objects), respectively.

On the contrary, the wormholes allow the random movement of objects towards the present best universe. According to the fundamental MVO algorithm, the $j^{th}$ variable of $i^{th}$ universe $w_j^i$ is denoted as follows.

$$w_j^i = \begin{cases} 
W_j + TDR + \left( (ub_j - lb_j) \cdot \text{rnd}_3 + lb_j \right), & \text{if} \text{rnd}_2 < 0.5 \\
W_j - TDR + \left( (ub_j - lb_j) \cdot \text{rnd}_3 + lb_j \right), & \text{if} \text{rnd}_2 \geq 0.5 \\
W_j^{\text{RouletteWheel}} & \text{if} \text{rnd}_1 \geq \text{WEP} \\
W_j^{\text{RouletteWheel}} & \text{if} \text{rnd}_1 < \text{WEP} 
\end{cases}$$

Here $\text{rn}_1, \text{rn}_2,$ and $\text{rn}_3$ are three random numbers in the range $[0,1]$. $W_j$ denotes the $j^{th}$ weight parameter where $W$ is the best weight vector. $ub_j$ and $lb_j$ denotes the upper and lower bound of the $j^{th}$ element. $W_j^{\text{RouletteWheel}}$ denotes the $j^{th}$ component selected by roulette-wheel technique. \text{WEP} denotes the wormhole existence probability, which is defined in Eq. (2).

$$\text{WEP} = \min + \text{current} \cdot \frac{\max - \min}{\text{Max}_{\text{iteration}}}$$

$TDR$ denotes another coefficient, namely travel distance rate which is defined in Eq. (3).
In the optimization process, TDR and WEP values determine the amount and frequency of updates made to a solution, respectively. The parameter $p$ in Eq. (3) represents the exploitation accuracy. A higher value of WEP always guarantees a higher degree of exploitation. We can achieve exploration by updating the current solution (i.e., black hole) by the current best solution (i.e., white hole) and this can be accomplished by the roulette wheel selection mechanism (to avoid local optima). We can carry out the optimization process by maintaining a balance between exploitation and exploration which is controlled by altering the TDR and WEP values, respectively.

2.2. **MV-MFLN-BASED forecasting**

In this subsection, we have presented the proposed MV-MFLN-based forecasting method. The input vectors are created by sliding a fixed-sized window over the exchange rate data. In each movement, a new data point of the vector is included and an old one is removed. We treat each vector generated in this way as a training pattern for the model. The training patterns are then normalized. Each normalized input signal of the vector passes through a block for functional expansion of $N$ number of trigonometric functions. We use basic sine and cosine trigonometric functions to carry out the expansion of the input vector. Therefore, after $N$ functional expansion of $d$ inputs, the dimension of input vector is $d*N$. The input layer receives the expanded inputs along with the original data points and computes the model output at the output layer.

In contrast to conventional FLN, the output neuron is a multiplicative unit that computes the weighted product of functionally expanded signals. We apply linear activation to the input neurons and sigmoid activation function at the output neuron. The product unit further increases the dimensions and thus provides a better chance to capture the nonlinearity existing in the input patterns. The target is presented at the output layer and compared with the model estimation. The difference obtained is treated as the error. The error thus obtained is backpropagated and is used for updating the neuron weights and biases. In this way, the network performs an input to output mapping. However, issues like early convergence and trapping to local minima etc., are associated with this gradient descent-based learning algorithm. Therefore, as mentioned earlier, we have used MVO for updating the neuron weights and biases. Fig. 1 presents the proposed model.

In MVO, the universe represents the vector of weights and the biases. The solution space comprises of several such universes. The search process is carried out by starting from a randomly selected instance of universes and is terminated upon reaching an optima. The solutions are updated as shown in Eq. (1). The MVO search process uses the exploration and exploitation capability in the search space during model training and finally terminated at the near-optimal weight and bias vector. As shown in Fig. 1, let $d$ be the number of features and $N$ be the number of functional expansion units. The input vector is of size $s = (d \times N)$ and the weight vector is of size $s + 1$ (bias). Let each element of the input pattern be $x(k), 1 \leq k \leq d$. Then let each element of input pattern $x(k)$ be functionally expanded as $z_k(n), 1 \leq n \leq N$. The output neuron computes the output and error as in Eq. (4) and Eq. (5).

$$\hat{y}(t) = \varphi \left( \prod_{i=1}^{N} \left( z_i(i) \times w_i(i) \right) + \text{bias} \right)$$

$$e(t) = y(t) - \hat{y}(t)$$

Here, $\varphi$ is the nonlinear activation function (sigmoid in this case). Based on the above discussion, the overall steps of MVO-MFLN-based forecasting method is depicted in Fig. 2.

3. **Experimental data analysis**

The exchange rate datasets are freely available in the internet hyperlink https://www.exchangerates.org.uk/. We have collected the data during the period 07th March 2019 and 05th September 2019 [39]. Table 1 presents the exchange rate data and other statistics associated with it. The present research focuses on the exchange rate prices of South Asian countries. The research data are associated with developed, developing, and underdeveloped countries. We see that the CNY/USD possesses the lowest mean, standard deviation, and variance, followed by INR/USD. For others, these values are found higher. Fig. 3 shows the six-time series datasets. From the Durbin and Watson test [40] (as in Table 2), it is observed that all the time series possesses no serial correlation. These pieces of evidence are in support of the autocorrelation function test (ACF). This test is used to detect the existence of autocorrelation at lag 1. The test statistics are positive and close to zero, which means there exists a weak serial correlation (the statistic always lies between

$$TDR = 1 - \left( \frac{current_{best}}{\text{Max}_{\text{iteration}}} \right)^{1/p}$$

$$e(t) = y(t) - \hat{y}(t)$$
0 and 4). Figs. 4–9 show the ACF plots of all the time series considered in this study. Fig. 10 shows the histograms of log-returns from six-time series against the theoretical normal distribution. These observations support the nonlinear and no serial correlation nature of the exchange rate time series. Hence, it is quite challenging to forecast these time-series. All the experiments in this study are carried out in MATLAB-2015, with Intel® core TM i3 CPU, 2.27 GHz processor, and 2 GB RAM.

4. Results and discussion

For performance analysis of the proposed model, we have developed and considered four promising comparative models such as GA-MFLN, DE-MFLN, TLB-MFLN, and GD-MFLN. The input datasets, normalization method, number of functional expansions (seven trigonometric functions), and activation functions are kept the same for all the models. We have conducted different experiments for (1, 5, 10, 15, and 30)-days-ahead forecasting of six exchange rate datasets. The mean absolute percentage error (MAPE) accuracy measure (as in Eq. (6)) is used for comparative performance analysis.

\[
MAPE = \frac{1}{N} \sum_{i=1}^{N} \left| \frac{\hat{x}_i - x_i}{x_i} \right| \times 100\% 
\]  

(6)

Here, “\(x_i\)” and “\(\hat{x}_i\)” represents the actual and estimated value of the exchange rate. “\(N\)” represents the total number of training data. We have considered a forecast as good if it generates a MAPE value closer to zero.

Tables 3–7 presents the MAPE values obtained from different simulations for multi-step-ahead forecasting. We assign the models with a rank value based on their MAPE value. Lower the MAPE value lower is the rank. The average rank in the table is assigned only in situations where a tie occurs. The last two columns of all tables show the average rank and re-ranking values for a model across the datasets. From 1-step-ahead forecasting (as in Table 3), we have observed that the MV-MFLN model provides the best rank in the case of four-time series, followed by TLB-MFLN and GA-MFLN each in one time series. The average rank
of MV-MFLN is one, and hence MV-MFLN is the best model among the other alternatives.

Similarly, in the case of 5-day-ahead prediction, the best MAPE is achieved by the MV-MFLN model in five time-series datasets. The proposed model earned 1st rank in 10-day-ahead, 15-day-ahead, and 30-day-ahead forecasting. Considering the five-time horizon and six datasets, the MV-MFLN achieved the best rank twenty-two times, the TLB-MFLN earned the best rank six times, and the GA-MFLN achieved four times. We find that the average performance of MV-MFLN is higher than other comparative models. In Fig. 11 shows the average MAPE of all the models considering six datasets and five horizons. We observe that the MV-MFLN and TLB-MFLN are the better performing models. Considering the results from Tables 3–7, we claim that the MV-MFLN model is the superior model as compared to the others.

Table 1
Statistics for exchange rate datasets.

<table>
<thead>
<tr>
<th>Exchange rate series</th>
<th>Abbreviation used</th>
<th>Minimum</th>
<th>Mean</th>
<th>Maximum</th>
<th>Standard deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Indian Rupee to US $</td>
<td>INR/USD</td>
<td>67.943</td>
<td>68.627</td>
<td>72.385</td>
<td>0.7741</td>
</tr>
<tr>
<td>Chinese Yuan to US $</td>
<td>CNY/USD</td>
<td>6.6872</td>
<td>6.8253</td>
<td>7.1397</td>
<td>0.0884</td>
</tr>
<tr>
<td>Bangladesh Taka to US $</td>
<td>BDT/USD</td>
<td>84.0813</td>
<td>84.2942</td>
<td>85.055</td>
<td>0.2625</td>
</tr>
<tr>
<td>Nepalese Rupee to US $</td>
<td>NPR/USD</td>
<td>108.3437</td>
<td>110.9313</td>
<td>114.3083</td>
<td>1.2766</td>
</tr>
<tr>
<td>Pakistan Rupee to US $</td>
<td>PKR/USD</td>
<td>138.8875</td>
<td>147.3721</td>
<td>163.542</td>
<td>8.2311</td>
</tr>
<tr>
<td>Sri Lankan Rupee to US $</td>
<td>LKR/USD</td>
<td>173.0057</td>
<td>175.6921</td>
<td>180.1725</td>
<td>1.4832</td>
</tr>
</tbody>
</table>
5. Conclusions

Exchange rate prediction is an exciting but complicated task. The exchange rate dataset possesses inherent dynamism and nonlinear characteristics, which make the input–output mapping difficult. Multilayer ANN and conventional gradient-based ANN learning algorithms have issues like structural complexity, early convergence, and trapping to local minima. This paper presents a hybrid model called MV-MFLN to get rid of the above issues. The model uses an MFLN model as the ANN model which has a product unit instead of a summation unit at the output layer. The product unit further amplifies the inputs, thus providing a better chance to capture the nonlinearity associated with it. MVO is used to fine-tune the weights and biases of the MFLN. The proposed model uses original data along with functionally expanded inputs as the model input. Prediction accuracies of MV-MFLN are assessed in terms of MAPE from six exchange rate time series. We have conducted different experiments for multi-step-ahead forecasting and compared the obtained results with other models such as GA-MFLN, DE-MFLN, TLB-MFLN, and GD-

<table>
<thead>
<tr>
<th>Exchange rate series</th>
<th>Durbin and Watson test value</th>
</tr>
</thead>
<tbody>
<tr>
<td>INR/USD</td>
<td>1.7447e-11</td>
</tr>
<tr>
<td>PKR/USD</td>
<td>9.3620e-12</td>
</tr>
<tr>
<td>BDT/USD</td>
<td>1.9664e-11</td>
</tr>
<tr>
<td>CNY/USD</td>
<td>9.4633e-12</td>
</tr>
<tr>
<td>NPR/USD</td>
<td>1.7286e-11</td>
</tr>
<tr>
<td>LKR/USD</td>
<td>9.4812e-12</td>
</tr>
</tbody>
</table>

Table 2: p-values from Durbin Watson tests for no serial correlation.

Fig. 3. Time series from six exchange rate datasets.
Fig. 4. ACF of INR/USD

Fig. 5. ACF of PKR/USD
Fig. 6. ACF of BDT/USD

Fig. 7. ACF of CNY/USD.
Fig. 8. ACF of NPR/USD

Fig. 9. ACF of LKR/USD
Table 3
MAPE statistics from 1-day-ahead forecasting.

<table>
<thead>
<tr>
<th>MODEL</th>
<th>MAPE</th>
<th>Average Rank</th>
<th>Re-ranking</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>INR/USD</td>
<td>CNY/USD</td>
<td>PKR/USD</td>
</tr>
<tr>
<td>GD-MFLN</td>
<td>0.06825(5)</td>
<td>0.07425(5)</td>
<td>0.06350(4)</td>
</tr>
<tr>
<td>GA-MFLN</td>
<td>0.05574(3)</td>
<td>0.05758(2)</td>
<td>0.04258(1)</td>
</tr>
<tr>
<td>DE-MFLM</td>
<td>0.06007(4)</td>
<td>0.07260(4)</td>
<td>0.06452(5)</td>
</tr>
<tr>
<td>TLB-MFLN</td>
<td>0.04657(2)</td>
<td>0.05765(3)</td>
<td>0.04651(3)</td>
</tr>
<tr>
<td>MV-MFLN</td>
<td>0.03945(1)</td>
<td>0.05336(1)</td>
<td>0.04279(2)</td>
</tr>
</tbody>
</table>

Table 4
MAPE statistics from 5-day-ahead forecasting.

<table>
<thead>
<tr>
<th>MODEL</th>
<th>MAPE</th>
<th>Average Rank</th>
<th>Re-ranking</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>INR/USD</td>
<td>CNY/USD</td>
<td>PKR/USD</td>
</tr>
<tr>
<td>GD-MFLN</td>
<td>0.06873(5)</td>
<td>0.07463(4.5)</td>
<td>0.06551(5)</td>
</tr>
<tr>
<td>GA-MFLN</td>
<td>0.05587(2)</td>
<td>0.05777(1)</td>
<td>0.06478(4)</td>
</tr>
<tr>
<td>DE-MFLM</td>
<td>0.06211(4)</td>
<td>0.07463(4.5)</td>
<td>0.06474(3)</td>
</tr>
<tr>
<td>TLB-MFLN</td>
<td>0.0560(3)</td>
<td>0.05785(2)</td>
<td>0.04688(2)</td>
</tr>
<tr>
<td>MV-MFLN</td>
<td>0.03996(1)</td>
<td>0.05831(3)</td>
<td>0.04471(1)</td>
</tr>
</tbody>
</table>
MFLN. The proposed model possesses low computational overhead and enhanced prediction accuracy. We can suggest it as a promising tool for exchange rate forecasting. Additionally, it can also be applied as a prediction model to other time-series datasets. In future, one may explore other higher-order neural networks and optimization techniques for improved forecasting accuracy.

### Table 5
MAPE statistics from 10-day-ahead forecasting.

<table>
<thead>
<tr>
<th>MODEL</th>
<th>MAPE</th>
<th>Average Rank</th>
<th>Re-ranking</th>
</tr>
</thead>
<tbody>
<tr>
<td>GD-MFLN</td>
<td>0.06892(5)</td>
<td>5</td>
<td>4</td>
</tr>
<tr>
<td>GA-MFLN</td>
<td>0.05699(3)</td>
<td>2.83</td>
<td>2</td>
</tr>
<tr>
<td>DE-MFLM</td>
<td>0.06231(4)</td>
<td>3.66</td>
<td>3</td>
</tr>
<tr>
<td>TLB-MFLN</td>
<td>0.05688(1)</td>
<td>1.75</td>
<td>1</td>
</tr>
<tr>
<td>MV-MFLN</td>
<td>0.05690(2)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

### Table 6
MAPE statistics from 15-day-ahead forecasting.

<table>
<thead>
<tr>
<th>MODEL</th>
<th>MAPE</th>
<th>Average Rank</th>
<th>Re-ranking</th>
</tr>
</thead>
<tbody>
<tr>
<td>GD-MFLN</td>
<td>0.10089(5)</td>
<td>5</td>
<td>5</td>
</tr>
<tr>
<td>GA-MFLN</td>
<td>0.05766(3)</td>
<td>2.75</td>
<td>3</td>
</tr>
<tr>
<td>DE-MFLM</td>
<td>0.06287(4)</td>
<td>4</td>
<td>4</td>
</tr>
<tr>
<td>TLB-MFLN</td>
<td>0.05760(2)</td>
<td>1.75</td>
<td>2</td>
</tr>
<tr>
<td>MV-MFLN</td>
<td>0.05705(1)</td>
<td>1.5</td>
<td>1</td>
</tr>
</tbody>
</table>

### Table 7
MAPE statistics from 30-day-ahead forecasting.

<table>
<thead>
<tr>
<th>MODEL</th>
<th>MAPE</th>
<th>Average Rank</th>
<th>Re-ranking</th>
</tr>
</thead>
<tbody>
<tr>
<td>GD-MFLN</td>
<td>0.17248(5)</td>
<td>5</td>
<td>5</td>
</tr>
<tr>
<td>GA-MFLN</td>
<td>0.05792(2)</td>
<td>3.16</td>
<td>3</td>
</tr>
<tr>
<td>DE-MFLM</td>
<td>0.06575(3)</td>
<td>3.5</td>
<td>4</td>
</tr>
<tr>
<td>TLB-MFLN</td>
<td>0.06580(4)</td>
<td>2.5</td>
<td>2</td>
</tr>
<tr>
<td>MV-MFLN</td>
<td>0.05744(1)</td>
<td>1.16</td>
<td>1</td>
</tr>
</tbody>
</table>

Fig. 11. Average MAPE from five models and five-time horizons.
References


