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Mass Spectra of Mesonic Molecules at Finite Temperature

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Abstract

At a finite temperature, exotic hadronic systems, mass spectrum was studied using the quark structure and the radial Schrödinger equation with the Cornell interaction potential representing the hadronic interaction. The projective unitary representation has been used in the study of hadronic ground states in physics. A relativistic bound state's mass spectrum with temperature dependence is shown to have a unique feature. The resulting values are compared to experimental and theoretical values, and the study showed a high level of conformity with additional values wherever feasible.

Keywords

bound state, exotic systems, mass spectrum, mesonic molecule, Schrödinger equation

Cover Page Footnote

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1. Introduction

It is essential to investigate the hadronic system's finite temperature dependence at great energy and intense contact. This connection holds for various possible models [1−4] and explains a previously unknown exotic hadronic bound state characteristic. The radial Schrödinger equation, and an analytic approach based on the correlation function behavior of hadronic cores in a finite temperature strong field, are used to depict interaction in temperature-dependent terms. The binding energy and bound state mass are calculated using this approach [5].

Exotic atomic and molecular states do not conform to more conventional states. They include states involving muons and pions and multiquark states. The exotic system is a multibody hadronic state investigated using various potential and framework techniques, including the microscopic cluster model, the quark-meson coupling model, and the Gaussian expansion method. Recent experimental studies of high energy hadron−hadron collisions have revealed many unusual states.

At finite temperatures, it has been demonstrated that the mass spectrum can be significantly larger than anticipated theoretically. The FINUDA Collaboration, SKS, the DANE machine, the PANDA Experiment, and KEK collected these experimental findings. Japan Proton Accelerator Research Complex (J-PARC) is projected to take on a new dimension and open up new study areas for hadronic physics at finite temperature in a brand-new operation in Japan. Thus, theoretical investigations can raise awareness and intrigue in experimental interpretations.

Exotic bound systems in the form of di-hadronic molecules are investigated in this research. A di-hadronic molecule (di-mesonic molecule or di-mesonic bound state) is a multiquarks state or a set of two mesons bound together by a strong force [6]. The exotic molecule states like BB, B_sB, D^*D, D_sB, K^*K^*, etc. are identified as mesonic molecule states or exotic mesonic molecules. B-meson contains anti-bottom, up, down, charm, and strange quarks:

$$\left(B^+ : u\overline{b}, B : d\overline{b}\, B_s : s\overline{b}\, B_c^+ : c\overline{b}\right)$$

and all of them have an anti-particle, which contains bottom, anti-up, anti-down, anti-charm, and anti-strange quarks:

$$\left(B^- : \overline{u}b\, \overline{B} : \overline{d}b, \overline{B_s} : \overline{s}b\, \overline{B_c^+} : \overline{c}b, \right).$$

Thus, a B_sB mesonic molecule can be presented as a multi quark exotic state $(B_sB : s\overline{b}u\overline{b})$.

The energy and mass spectrum of the di-hadronic molecules two-body systems with Cornell potential in the lower and higher states are determined using the correlation functions' Gaussian asymptomatic behavior. Furthermore, a relativistic correction to the component core mass is derived.

The Schrödinger equation and the constituent meson component's mass calculate the hadronic molecule's (constituent meson component's) mass spectrum. The exotic system mass component is defined via modifying correction. Cluster models are effective in describing hadronic bound state masses [7,8]. Consequently, the hadronic bound system is investigated concerning the quarks constituent system at a finite temperature; this notion, in conjunction with the radial Schrödinger equation, determines the hadronic characteristic [9,10].

The studies propose that the charged component particles of hadronic cores $\{(\text{hadron})_1$ and $(\text{hadron})_2\}$ at finite temperature, particularly quarks, may also be utilized to define the new property of hadronic masses, providing a plausible approximation for describing the features of hadronic systems in both strong and weak interactions. The mass of a hadronic-bound state in a confining potential at a finite temperature is calculated in this paper [11]:

$$V(x,T) = A_1(x,T)x + A_2(x,T)x^2 + A_3(x,T)x^{-1} + A_4(x,T)x^{-2}. \tag{1}$$

where x is the effective radius of the bound state, T is the temperature and A_i are parameters.

As per the projective unitary representation (PUR) [12,13]. The Hamiltonian of hadronic bound state $(h = c = 1)$ is defined as:

$$\widehat{H}R(\widehat{x}) = E_\ell(\mu, T)R(\widehat{x}).$$

$$\widehat{H} = \frac{\widehat{p}^2}{2\mu_1^2} + \frac{\widehat{p}^2}{2\mu_2^2} + A_1(x,T)x + A_2(x,T)x^2 + A_3(x,T)x^{-1} + A_4(x,T)x^{-2}. \tag{2}$$

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or can be expressed as

$$\ddot{R}(\hat{x})+\frac{2\dot{R}(\hat{x})}{\hat{x}}-\frac{\ell(\ell+1)}{2\hat{x}^2}-A_1(x,T)x- \\ -A_2(x,T)x^2-A_3(x,T)x^{-1}-A_4(x,T)x^{-2}+ \\ +E_\ell\left(\frac{\sqrt{m_2^2-2\mu^2\dot{E}_\ell}\cdot\sqrt{m_1^2-2\mu^2\dot{E}_\ell}}{\sqrt{m_1^2-2\mu^2\dot{E}_\ell}+\sqrt{m_2^2-2\mu^2\dot{E}_\ell}}\right)=0, \tag{3}$$

where μ_1, μ_2 indicate the component mass (constituent mass) of hadronic cores in the bound system, distinct from the remaining rest masses m_1 and m_2. The components masses of the parameters are given below.

We assume $A_2(x,T) = A_4(x,T) = 0$, and refer to it as a Cornell potential in the previous equation to simplify the calculation. As an exponential function represents the nonzero temperature following the Debye mass, the given function may be changed as follows using approximation limits:

$$A_1(x,T)\approx -a(T)m_D x e^{-m_D x}\simeq -a(T)m_D x, \\ A_3(x,T)\approx c(T)e^{-m_D x}\simeq \\ \simeq 0.5c(T)m_D x^2 - c(T)x m_D, \tag{3*}$$

where $a(T)$, $c(T)$, $b(T) = 0.5c(T)$ are parameters and m_D is the Debye mass, equal to zero at absolute zero (T_0), or can be expressed as follows [12]:

$$A_1(x,0)\approx a(T_0), \quad A_3(x,0)\approx c(T_0).$$

Additionally, the Debye mass of mesons is denoted by the following value [12]:

$$m_D(T)\cong 11.025Ta(T)=\alpha T \tag{3**}$$

where

$$0<T<130MeV$$

and $\alpha = 11.025a(T)$ is a parameter.

The radial Schrödinger equation has to be changed to solve and explain the exotic hadrons experiment findings [13]. By considering renormalization plus the nonrelativistic limit, the modified equation yielded the interaction Hamiltonian in quantum field theory formalism, the scattering matrix, and the corresponding Feynman diagram [4,14]. The correlation function between the field's associated current and the quantum numbers calculates the particle's mass. It is related to the Feynman functional path integral (FFPI) and n-point Green's function in quantum mechanics at the nonrelativistic limit [13,15].

Consequently, the bound hadronic state mass is denoted as the correlation function's asymptotic limit. Therefore, the system's mass is calculated as follows:

$$M=\min_{\mu_1,\mu_2}\left(E_\ell(\mu,T)+\frac{\mu_1+\mu_2}{2}+\frac{m_1^2\mu_2+m_2^2\mu_1}{2\mu_1\mu_2}\right) \\ =\mu_1+\mu_2+\mu\dot{E}_\ell(\mu,T)+E_\ell(\mu,T). \tag{4}$$

and

$$\mu_1=\sqrt{m_1^2-2\mu^2\dot{E}_\ell(\mu,T)}. \\ \mu_2=\sqrt{m_2^2-2\mu^2\dot{E}_\ell(\mu,T)}, \tag{5}$$

where $E_\ell(\mu,T)$ is the eigenvalue of the total Hamiltonian and

$$\dot{E}_\ell(\mu,T)=\frac{\partial E_\ell(\mu,T)}{\partial\mu},$$

The μ parameter represents the boundary system's component mass.

1.1. Research aim

In this study, the exotic mesonic is defined in this theoretical research as the Schrödinger equation solution of a mesonic molecule-bound state in the Cornell potential at finite temperature $(0.01MeV\le T\le 5MeV)$. This opinion was formed using the PUR technique. As is apparent, the quarks' behavior in their exotic states near their deconfinement temperature is essential in settings with high energy interactions, including quark stars, strange stars, and compact stars. Calculations using different models often provide imprecise results, and we cannot forecast the exact mass value. The exotic particles' mass spectrum must be predicted using relativistic behavior. Consequently, the bound state mass spectrum at a given temperature is presented using quantum field theory, and a connection between the mass spectrum with temperature is established.

2. Materials and methods

2.1. Projective unitary form of the Schrödinger equation

The behavior of the bound states and the relativistic corrections to the Hamiltonian in a strongly interacting environment at the finite temperature or near the deconfinement (particle is free and move relatively independently) and confinement temperature are largely unknown that various model predicts the mass with temperature variation. The temperature-dependent modified radial Schrödinger equation (MRSE) for Cornell potential is studied in different ways and manners. In this article, the MRSE for the linear plus inverse Coulombic potential at finite temperature is solved (i.e., Cornell type potential). The RSE is analytically solved using the PUR method.

In a theoretical and analytical manner, the projective unitary technique is one of the effective methods for solving the RSE. Therefore, the inclusion of the relativistic bound state at finite temperature is one of the most interesting subjects of contemporary theoretical high-energy physics. This research is devoted to investigating the main subject of bound states at finite temperature. Thus, the mass spectrum of the constrained state (bound states) is determined by the asymptotic behavior (at $x\to\infty$ limit) of the correlation functions (CF) of the corresponding currents of the charged particle over quantum distributions with the exact quantum numbers. The CF and mass spectrum which are described in terms of the FFPI and n-point Green's function method in the $A_\mu\ (x)$ external field can be done thoroughly. As we know, the FFPI allows us to allocate the required asymptotic manner of CF. The resulting data are similar to the FFPI in Equations (4) and (5) in the formalism of nonrelativistic quantum mechanics. Hence, the inter-hadronic potential is determined by the Feynman diagram, the resulting exchange of the $A_\mu\ (x)$, and the mass in the RSE is the constituent differing from the mass of the rest state of the hadronic bound system [14]. Thus, thanks to the building block mass of components (mesons) in the bounding system and the effective inertial mass that are defined, we can determine the relativistic modifications and corrections to the strong interconnection Hamiltonian at the finite temperature.

The RSE for the exotic meson bound state has linear and nonlinear components that describe the possibility of interactions between clusters (hadronic cores) at the nonzero temperature reads:

$$\left[\frac{\hat{p}^2}{2\mu}+A_1(x,T)x+A_2(x,T)x^{-1}\right]R(x) = E_\ell(\mu,T)R(x), \tag{6}$$

Then a correction to the constant parameters in Equation (1) and make the potential interaction part temperature-dependent in Equation (3*) and (3**), Equation (6) reads as follows when the Debye mass is used:

$$\left[\frac{\hat{p}^2}{2\mu}+a(T)\alpha T\,x^2+b(T)\alpha T-c(T)\alpha T\right]R(x) = E_\ell(\mu,T)R(x). \tag{7}$$

Quantum field theory describes systems with finite temperature as a limitless number of oscillators maintaining their oscillating characteristics throughout interactions. To use quantum field techniques [14], must be replaced in the equation for the Cornell potential's linear and nonlinear contact components (4).

From Equation (2), we will define the eigenenergy, mass spectrum, and wave function by applying the PUR method [13]. The PUR technique is generally used to define and determine the eigenenergy value, energy levels, and wavefunction of the restricted mesonic states or multi-body bound states. The wavefunction of the exotic mesonic molecules has the form:

$\psi(\hat{r})=\hat{r}^\ell Y_{\ell m}(\theta,\varphi)R(\hat{r})$.

Now let us consider large distances. Usually, we can find the asymptotic behavior of the wave function $R(\hat{x})$ for $\hat{x}\to\infty$ analytically. Let this asymptotic be

$$\hat{x}=\hat{q}^{2\rho}\to R(\hat{r})=\hat{q}^{2\rho\ell}\Phi(\hat{q}^2), \tag{8}$$

where ρ is a parameter and for the class of Coulomb potential $\rho\ =1$, for Carnell potential $\rho\sim 0.5$.

When more considerable distances are involved, the wave function must have a Gaussian solution; therefore, the Hamiltonian is used with the PUR variables from Equations (8) and (7). In a new auxiliary space $(\hat{q}^{2\rho})$, Equation (7) is obtained:

$$\left[\frac{\hat{p}_q^2}{2}+4\mu\rho^2\hat{q}^{4\rho-2}\{-a(T)\alpha T\hat{q}^{4\rho}+b(T)\alpha T\hat{q}^{2\rho}-c(T)\alpha T-E_\ell(\mu,T)\}\right]\Phi=0, \tag{9}$$

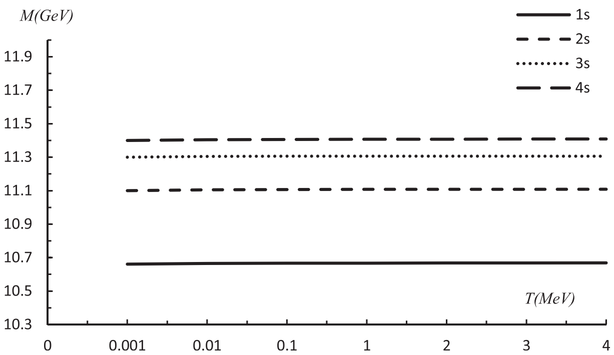

Fig. 1. The mass spectra of $\boldsymbol{B\overline{B}}$ an exotic mesonic molecule in the $\boldsymbol{n}=1,2,3$, $\ell=0$ levels as a function of T in $\boldsymbol{MeV}$. $0.151 \le \boldsymbol{a} \le 0.189(\boldsymbol{GeV})^2$, $0.001 \le \boldsymbol{c} \le 0.003$, $0.01\boldsymbol{MeV} \le \boldsymbol{T} \le 5\boldsymbol{MeV}$, and $\boldsymbol{m_B} = 5.280\boldsymbol{GeV}$.

where

$$\widehat{q} = \frac{\widehat{a}^- + \widehat{a}^+}{\sqrt{2\omega_\ell}}, \quad \widehat{p}_q^2 = \sqrt{\frac{\omega_\ell}{2}}\,\frac{\widehat{a}^- - \widehat{a}^+}{i}, \tag{10}$$

where ω_ℓ is the pure oscillator frequency, ℓ is the orbital angular momentum quantum number, the creation operator is $(\widehat{a}^+)$, the annihilation operator is $(\widehat{a}^-)$, and $\widehat{q}, \widehat{p}$ are the oscillator canonical variables, can be written in the form of operators ($\widehat{a}^-$), ($\widehat{a}^+$). Based on the PUR condition, the corresponding canonical variables are derived as Wick orderings [15−18]:

$$\begin{aligned} \widehat{a}^- &= \sqrt{\frac{\omega}{2}}\left(\widehat{q} + \frac{i}{\omega_\ell}\widehat{p}_q\right), \\ \widehat{a}^+ &= \sqrt{\frac{\omega}{2}}\left(\widehat{q} - \frac{i}{\omega_\ell}\widehat{p}_q\right), \end{aligned} \tag{11}$$

and

$$\begin{aligned} \widehat{p}_q^2 &= \frac{(2+2\rho+4\rho\ell)}{2}\omega_\ell + :\widehat{p}_q^2:. \\ \widehat{q}^2 &= (2+2\rho+4\rho\ell)\frac{1}{2\omega_\ell} + :\widehat{q}^2:. \end{aligned} \tag{12}$$

The interaction Hamiltonian includes all non-square components of the term $:\widehat{q}^{2n}:, :\widehat{p}_q^{2n}:$ (a condition in Wick ordering), so we can then discover the renormalization of the bound state parameters, including the wave function, which enables us to introduce the PUR using the zero approximation and afterward find the eigenvalue of the ground state energy $\epsilon_0(E_\ell,\ T)$. Therefore, the following Equation (6) is constructed based on Equation (8) (for a more thorough explanation, see Ref. [13]):

$$\begin{aligned} \epsilon_0(E_\ell, T) &= \\ &= \frac{\widehat{p}_q^2}{2} + 4\mu\rho^2\widehat{q}^{4\rho-2}\{-a(T)\alpha T\widehat{q}^{4\rho}b(T)\alpha T\widehat{q}^{2\rho}\} \\ &\quad -4\mu\rho^2\widehat{q}^{4\rho-2}\{-E_\ell(\mu, T) + c(T)\alpha T\}. \end{aligned} \tag{13}$$

The following Equation (13) may therefore be rewritten:

$$\epsilon_0(E_\ell, T) = X(T, \omega_\ell) - E_\ell Y(T, \omega_\ell) = 0.$$

and then, using PUR conditions, we determine

$$\epsilon_0(E_\ell, T) = 0, \quad \frac{d\epsilon_0(E_\ell, T)}{d\omega_\ell} = 0, \tag{14}$$

The bound system's minimum ground state energy may now be determined using zero approximation [13]. Therefore:

$$\begin{aligned} E_\ell(\mu, T) = {}& \frac{(1.5+\ell)}{2\mu}\omega_\ell - (1.5+\ell)a(T)\alpha T\omega_\ell^{-1} \\ &+ \sqrt{\frac{(1.5+\ell)}{2}}b(T)\alpha T\omega_\ell^{-0.5} - c(T)\alpha T, \end{aligned} \tag{15}$$

and

$$\begin{aligned} &\Omega_\ell^4 - \frac{b(T)\mu\alpha T}{\sqrt{0.5(1.5+\ell)}}\Omega_\ell + 2a(T)\mu\alpha T = 0. \\ &\dot{E}_\ell(\mu, T) = \frac{\partial E_\ell(\mu, T)}{\partial\mu} = -\frac{(1.5+l)}{2\mu^2}\Omega_\ell^2. \end{aligned} \tag{16}$$

where $\Omega_\ell = \sqrt{\omega_\ell}$ is a new parameter.

Therefore, using Equations (4) and (16), the mass spectra of the constrained states may be calculated. The findings may now be utilized to compute the bound state mass and binding energy via a fully modified radial Schrödinger equation. Total Hamiltonian is used for calculation simplicity regardless of spin−spin, spin-orbital, or tensor−tensor interactions and their effects at finite temperature states.

3. Hadronic exotic constrained state

In this method, PUR [12] is added to the eigenenergy and mass spectra of the constrained state. Consequently, in nuclear physics, the correctness of PUR may be determined by comparing it to the findings of other computations. If the hadronic core's mass is suitably large, the hadronic system's energy spectrum may be defined. Consequently, the systems' energy spectrum may be calculated via utilizing PUR and the quantum field theory. In this instance, the interaction

Table 1
Mass spectra of exotic hadronic systems.

Result of this work (GeV)						
M	$B\overline{B}$	$B_sB_s^*$	$B^*\overline{B}^*$	$B_s^*\overline{B}_s^*$	$B\overline{B}^*$	B_sB
$\ell=0,\ T\approx 0.01$ MeV	10.660	10.881	10.714	10.978	10.652	10.649
$\ell=1,\ T\approx 1$ MeV	11.172	11.214	11.146	11.345	11.024	11.015
$\ell=1,\ T\approx 5$ MeV	11.715	11.937	11.815	12.021	11.867	11.907
Results of other works (GeV), $T=0$						
M [26]	10.56	–	10.65	–	10.61	–
M [27]	–	–	10.648	–	10.602	–
M [28]	10.51		10.603	–	10.56	10.62
M [29]	10.410	10.690	10.494	10.771	10.542	10.594

potential is a Cornell type, and the mass and binding energy spectrums are produced by multiplying Equation (16) by the component mass given from Equation (5) [19–21]. Afterward, the bound parameters, which are comprised of $(hadron)_1$ and $(hadron)_2$ are found.

Numerous writers calculated the mass and the binding energy of hadronic systems at a finite temperature using various nuclear physics techniques based on possible phenomenological models and field methods. In these techniques, the cluster masses are selected as free parameters. The Particle Data Group-2020 [22] provides the following experimentally confirmed limitations on hadron masses:

$$m_B = 5.280GeV, \quad m_{B^*} = 5.324GeV,$$

$$m_{Bs^*} = 5.415GeV, \quad m_{B_s} = 5.366GeV$$

Additionally, the constant temperature parameters are determined from experimental data collected across the following range:

$$0.15 \le a \le 0.19(GeV)^2$$
$$0.002 \le c \le 0.003$$

ground state, and

$$0.15 \le a \le 0.19(GeV)^2$$
$$0.001 \le c \le 0.002$$
$$1MeV \le T \le 5MeV$$

in the excited states' starting states and the lack of spin–orbit interactions.

The bound system's component mass [23–25] and binding energy were determined via the rest mass plus orbital quantum numbers.

The mesonic molecules' mass spectrum in the $\ell = 0, 1,$ for $T = 0.01, 1, 5\ MeV$, is described numerically in Table 1, and then to assess schematically the effect of the finite temperature, the plot of the mass spectra of the $B\overline{B}$ states in the $n = 1, 2, 3, 4$ for $\ell = 0$ are presented in Fig. 1.

4. Conclusion

The Schrödinger equation's bound state solution for hadronic molecules at a finite temperature and a Cornell potential is determined in this paper utilizing the projective unitary representation. We characterized the temperature dependency of heavy mesonic molecules, including $B\overline{B}B_sB_s^*$ $B\overline{B}^*$ B_sB, $B_s^*\overline{B}_s^*$ and $B^*\overline{B}^*$. In high-energy contacts, the relativistic mass-temperature connection was developed. At zero and finite temperatures, we have characterized the heavy exotic compounds' mass spectrum.

Temperature dependency was found by treating nonzero temperature as an exponential function and modifying the Debye mass. The findings establish that mesonic atoms mass spectra with nonzero and zero temperatures may represent a novel property of hidden characteristics. According to the findings, the study's theoretical results are expected to open new avenues for new theoretical knowledge of exotic bound systems due to the work's outstanding and similar results to previous theoretical or experimental studies.

The theoretical data gained may be used in a future study, potentially opening up new possibilities for identifying unique characteristics of unusual hadronic systems. We showed that the temperature dependence of exotic molecule mass in Table 1 was significantly different in the ground state at $T = 0,\ 0.01MeV$ and in the more excited states at $1MeV \le T \le 5MeV$. The temperature-dependent will be enhanced by raising the orbital quantum number ℓ, which is also based upon

PUR. We have accurately calculated that the component particles' mass in mesonic molecules rises as the temperature increases.

Nevertheless, based on equations (15) and (16), we anticipate that at very high temperatures $T \to \infty$, the connection between temperature and bound state mass must change fundamentally, i.e., exotic hadronic molecular "bound states" cannot exist. Our findings:

i) A procedure for calculating the relativistic correction associated with the relative motion and strong interaction of mesonic molecules at finite temperature in a system has been elaborated. The relativistic corrections for mass have been determined analytically based on QFT and PUR methods. The use of methods and idea of the PUR in solving quantomechanical problems, i.e., the modified Hamiltonian in normal form (standard form) in terms of the $\hat{a}^+, \hat{a}^-$ operators, allows us to evaluate the mass spectrum for the lower and higher energy levels.

ii) The temperature-dependent MRSE is described by using the Cornell potential and the PUR approach. The mathematical statement for the constrained mass at finite temperature is presented. Results are used for describing temperature dependence mass spectra of exotic di-mesonic molecules. Consequently, studying for an analytical solution of the MRSE for the well-known potential within the substructure of QFT could present and provide helpful information of exotic bound states. The temperature-dependent MRSE opens a new perspective for further researches and confirmation. Authors can conclude that the theoretical outcomes of the bound state are expected to enable new possibilities of the exact and more general nature of the results at very high temperatures.

iii) We have been presented the numerical outcomes for some exotic mesonic molecules in Fig. 1, and we have seen that the present potential model for describing the mass of the mesonic molecule at zero and nonzero temperatures are quite a good candidate for hadronic physics in a brand-new operation in FINUDA, DANE, PANDA, KEK experiments. Thus, this theoretical investigation can raise awareness and intrigue in experimental interpretations.

Finally, we can deduce that the outcomes of this theoretical study in the form of the PUR approach will enable us to solve relativistic quantum problems, including the relativistic effect of mass and relativistic effect on the bound states in High Energy Physics based on the relativistic Schrödinger equation.

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Declaration of competing interest

The authors declare that there is no conflict of interest.

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