



Investigating the Nuclear Properties of 162-172 Er Isotopes using IBM-1, SEF, and NEE

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Keywords

Electromagnetic transitions, Energy levels, Er isotopes, IBM-1 model, NEE, SEF

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RESEARCH PAPER

Investigating the Nuclear Properties of $^{162-172}\text{Er}$ Isotopes Using IBM-1, SEF, and NEE

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Abstract

The energy levels of the ground state band (GSB), β , and γ -bands for $^{162-172}\text{Er}$ isotopes are calculated in this work by adopting the Interacting Boson Model (IBM-1), the Semi-Empirical Formula (SEF) and the New Empirical Equation (NEE). The GSB, β , and γ -bands results revealed that IBM-1, SEF, NEE, and the available experimental data are all in agreement with certain variations. The NEE is more compatible with the experimental data than the IBM-1 and SEF calculations. This study demonstrates that the SEF and NEE equations are able to describe the energy spectra of Er isotopes in comparison to IBM-1. The Er isotopes exhibit rotational SU(3) transition behavior.

Keywords: Electromagnetic transitions, Energy levels, Er isotopes, IBM-1 model, NEE, SEF

1. Introduction

The nucleus is a quantum-mechanical system composed of a specific number of interacting protons and neutrons identified as nucleons. The even–even nuclei are the most stable and abundant. Their stability increases significantly when the number of protons and neutrons, or both, is equal to one of the magic numbers [1–4]. The analysis of deformed nuclei in the rare-earth midshell region, such as Er isotopes, is essential to comprehending the entire quantum many-body system.

Several nuclear models have been developed to study nuclear structure, including the collective model, the IVBM model [5], and IBM-1. The IBM-1, which was introduced by Arima and Iachello [6], is one of the nuclear models that can accurately figure out the energy levels of even–even nuclei. The motion of nucleons outside the closed shell is modeled in this model as bosons, which are pairs of protons (N_π) and neutrons (N_γ). Bosons are calculated using the adjacent closed shell. If the shell is more than half full, the number of bosons equals the number of hole pairs. The nucleus is then

considered as a system of bosons ($N_b = N_\pi + N_\gamma$) [7–9]. The transition energy, energy levels, electromagnetic transition probabilities, potential energy, and quadrupole moment for 100-102Ru, 184 W, and 184Os nuclei were thoroughly studied [10,11]. The Er isotopes have been the subject of many studies. Abood and Al-Jubbori [14] used the IBM-2 model to calculate the energy levels and electromagnetic transition probabilities of $^{158-168}\text{Er}$ isotopes. They also determined the Hamiltonian for these isotopes. The nuclear deformation of Er–Os isotopes at $N = 102$ using the NEE and the IBM-1 model was calculated by Al-Jubbori et al. [12]. Kassim et al. [13] investigated the nuclear structure and energy levels of even–even ^{158}Er using IBM, and IVBM models. The number of neutrons significantly influences the level structure of Er isotopes. Extensive investigations have been carried out into the collective nature of these isotopes. The main objective of this paper is to demonstrate how the SEF and NEE equations are capable of addressing the energy spectra of Er isotopes when compared to IBM-1. as well as to establish the GSB, β , and γ bands for $^{162-172}\text{Er}$ isotopes using the IBM-1 model, SEF, and NEE methods.

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2. Theoretical section

2.1. IBM-1, SEF, and NEE calculations

The most important aspect of the IBM-1 model is that bosons correspond to correlated nucleon pairs while the remaining nucleons form the inert core of the nucleus. Each boson can occupy one of two levels, determined by its angular momentum: s-boson ($I=0$) or d-boson ($I=2$). The bosons interact to form the Hamiltonian tensor, then the IBM-1 Hamiltonian can be written in multipolar form as [14–19].

$$H = \varepsilon \hat{n}_d + a_0 \hat{P} \cdot \hat{P} + a_1 \hat{L} \cdot \hat{L} + a_2 \hat{Q} \cdot \hat{Q} + a_3 \hat{T}_3 \cdot \hat{T}_3 + a_4 \hat{T}_4 \cdot \hat{T}_4, \quad (1)$$

where ε is the boson energy. The operators are described as

$$\left. \begin{aligned} \hat{n}_d &= (d^\dagger \cdot \tilde{d}) \\ \hat{p} &= 1/2[(\tilde{d} \cdot \tilde{d}) - (\tilde{s} \cdot \tilde{s})] \\ \hat{L} &= \sqrt{10}[d^\dagger \times \tilde{d}]^1 \\ \hat{Q} &= [d^\dagger \times \tilde{s} + s^\dagger \times \tilde{d}]^{(2)} + \chi[d^\dagger \times \tilde{d}]^{(2)} \\ \hat{T}_r &= [d^\dagger \times \tilde{d}]^{(r)} \end{aligned} \right\}, \quad (2)$$

where \hat{n}_d : d-boson number operator, \hat{p} : pairing operator, \hat{L} : angular momentum operator, \hat{Q} : quadrupole operator, \hat{T}_n operator assigns $r=3$ and 4 to the octupole and hexadecapole operators, respectively. The dynamic symmetry of the IBM-1 is determined by the unitary group $U(6)$, which is reduced into three sub-chains: $U(5)$ for vibrating nuclei, $SU(3)$ for rotating nuclei, and $O(6)$ for γ -unstable nuclei as [20–23].

$$U(6) \supset \left\{ \begin{array}{l} nU(5) \supset O(5) \\ nSU(3) \\ nO(6) \supset O(5) \end{array} \right\} \supset O(2) \quad (3)$$

The properties of even–even nuclei can be predicted by calculating the energy ratio $R = \left(\frac{E(4_1^+)}{E(2_1^+)} \right)$, where $E(4_1^+)$ is the energy of the second excited level and $E(2_1^+)$ is the energy of the first excited level. due to the difficulty of studying the nuclear structure, particularly distorted nuclei. Several equations have recently been developed, including the Semi-Experimental Empirical Formula (SEF) [24] and the New Empirical Equation (NEE) [25]. The SEF formula for GSB is written as follows

$$E(I) = A_1 [e^{A_2 I} - A_3], \quad (4)$$

A_1 , A_2 , and A_3 were fitted to the measured energies of GSB, where A_1 , A_2 , and A_3 correspond to fitting parameters. The γ and β bands are computed as

$$E(I) = E_0 + (A_1 + B) [e^{A_2 I} - A_3] \quad (5)$$

The E_0 and B were fitted to the data for the γ and β bands. Similarly, the NEE formula was used to calculate all parameters required for calculations. For the GSB band

$$E(I) = \frac{A_1 I(I+1)}{A_2 (I+1) + IA_3}, \quad (6)$$

and for the γ and β bands

$$E(I) = E_0 + \frac{(A_1 + B)I(I+1)}{A_2 (I+1) + IA_3} \quad (7)$$

3. Results and discussion

The following is a discussion of the calculation of the GSB, γ and β bands for $^{162-172}\text{Er}$ isotopes using the IBM-1, SEF, and NEE frameworks:

As a preliminary step to the calculations, the ratio ($R_{4/2}$) is determined for each isotope. Preliminary results from Table 1 show that the $^{162-172}\text{Er}$ isotopes correspond to the deformed rotor $SU(3)$. The $R_{4/2}$ values of the low-lying energy levels of Er isotopes vary as a function of mass number. Table 1 clearly shows the transition occurred at the $SU(3)$ limit. We can clearly see that as the number of neutrons increases, the energy ratio also increases very slowly. Furthermore, the values of the IBM-1, α_2 and β_2 coefficients for $^{162-172}\text{Er}$ isotopes are shown in Table 2. The estimated GSB, β , and γ -band parameters for the SEF and NEE formulas for the subjected isotopes are shown in Tables 3–5, respectively.

The difference between IBM, SEF, and NEE energy levels and experimental data was calculated by root-mean-square-deviation (RMSD) [26].

$$RMSD = \left[\frac{1}{N} \sum_{i=1}^N |(E_{cal} - E_{exp})_i|^2 \right]^{1/2}$$

The RMSD values for the GSB, β and γ bands are listed in Table 6. The experimental values [27–32] for GSB, β , and γ -bands were compared to the IBM1, SEF, and NEE calculations as illustrated in Figures 1–3, respectively. The SEF and NEE

Table 1. The $R_{4/2}$ values for $^{162-172}\text{Er}$.

Isotopes	^{162}Er	^{164}Er	^{166}Er	^{168}Er	^{170}Er	^{172}Er
$R_{4/2}$	3.230	3.276	3.288	3.309	3.310	3.314

Table 2. IBM-1 parameters for Er isotopes.

IBM-1					
Isotopes	N_b	ELL	QQ	α_2	β_2
^{162}Er	13	0.026	-0.021	0.1153	-0.1522
^{164}Er	14	0.023	-0.019	0.1125	-0.1485
^{166}Er	15	0.0208	-0.0162	0.1090	-0.1439
^{168}Er	16	0.020	-0.015	0.1023	-0.1350
^{170}Er	17	0.0197	-0.017	0.0962	-0.1270
^{172}Er	18	0.0193	-0.0168	0.0303	-0.0400

ELL = $2a_1$; QQ = $2a_2$.

Table 3. SEF and NEE parameters of GSB for Er isotopes.

Isotopes	SEF [24]			NEE [25]		
	A_1	A_2	A_3	A_1	A_2	A_3
^{162}Er	2.825	0.049	1.062	0.098	4.264	-4.018
^{164}Er	1.679	0.069	1.093	0.061	3.611	-3.527
^{166}Er	1.265	0.076	1.089	0.071	4.587	-4.466
^{168}Er	0.815	0.102	1.106	0.055	3.927	-3.886
^{170}Er	1.247	0.081	1.113	0.051	3.707	-3.667
^{172}Er	1.238	0.080	1.112	0.051	3.722	-3.679

Table 4. SEF parameters for β and γ -bands in MeV.

Isotopes	N_b	β -band		γ -band	
		E_0	B	E_0	B
^{162}Er	13	1.020	-1.643	0.677	0.104
^{164}Er	14	1.259	-0.444	0.785	-0.180
^{166}Er	15	1.498	-0.405	0.636	-0.024
^{168}Er	16	1.242	-0.338	0.681	-0.002
^{170}Er	17	0.907	-0.201	0.782	-0.046
^{172}Er	18	—	—	0.875	-0.259

Table 5. NEE parameters for β and γ -bands in MeV.

Isotopes	N_b	β -band		γ -band	
		E_0	B	E_0	B
^{162}Er	13	1.014	-0.051	0.751	-0.0008
^{164}Er	14	1.229	-0.012	0.841	-0.008
^{166}Er	15	1.495	-0.022	0.700	-0.004
^{168}Er	16	1.216	-0.015	0.741	-0.003
^{170}Er	17	0.904	-0.008	0.844	-0.0002
^{172}Er	18	—	—	0.884	-0.003

values for GSB are illustrated in Figure 1. The agreements between the SEF, NEE, and IBM-1 calculations with the available experimental data are

Table 6. The root mean square deviation values for the GSB, β , and γ bands.

Isotopes	GSB			β -band			γ -band		
	IBM-1	SEF	NEE	IBM-1	SEF	NEE	IBM-1	SEF	NEE
^{162}Er	0.2284	0.0830	0.0376	0.3870	0.0236	0.0163	0.2242	0.0460	0.0178
^{164}Er	0.1249	0.0660	0.0093	0.3736	0.0528	0.0152	0.1721	0.0638	0.0523
^{166}Er	0.0838	0.0532	0.0116	0.1362	0.0485	0.0532	0.0543	0.0281	0.0042
^{168}Er	0.0287	0.0443	0.0038	0.1713	0.0224	0.0000	0.0269	0.0211	0.0016
^{170}Er	0.0437	0.0597	0.0051	0.1511	0.0562	0.0205	0.1356	0.0451	0.0218
^{172}Er	0.0572	0.0616	0.0058	—	—	—	0.0273	0.0033	0.0012

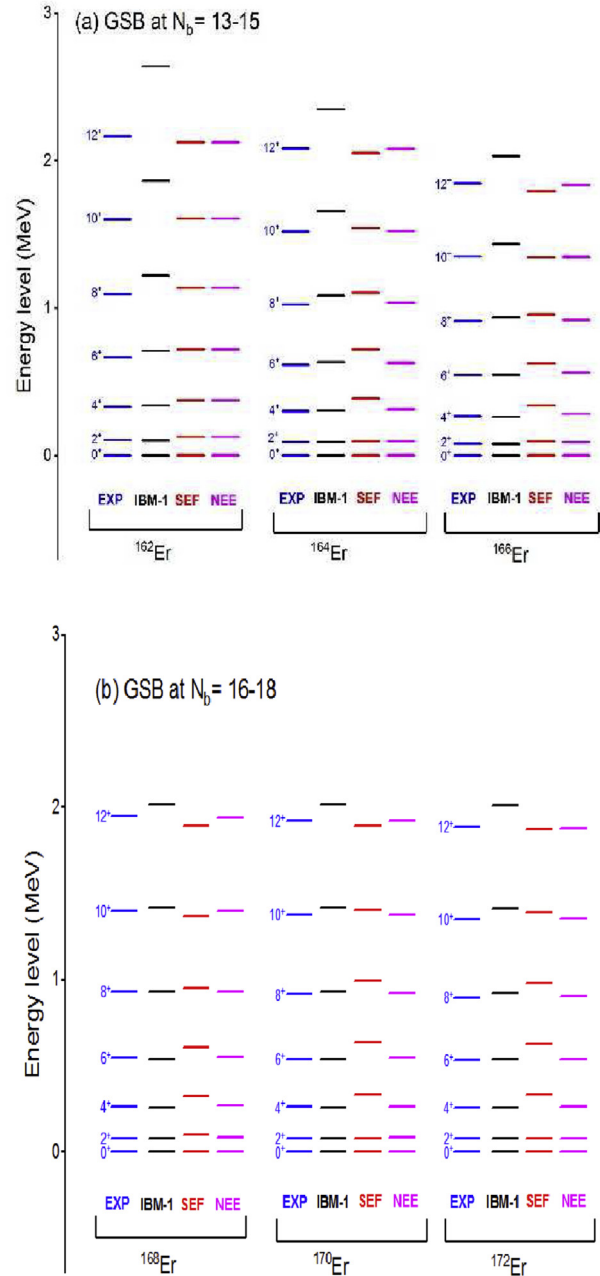


Fig. 1. The calculated and experimental values [27–32] of GSB for the $^{162-172}\text{Er}$ isotopes.

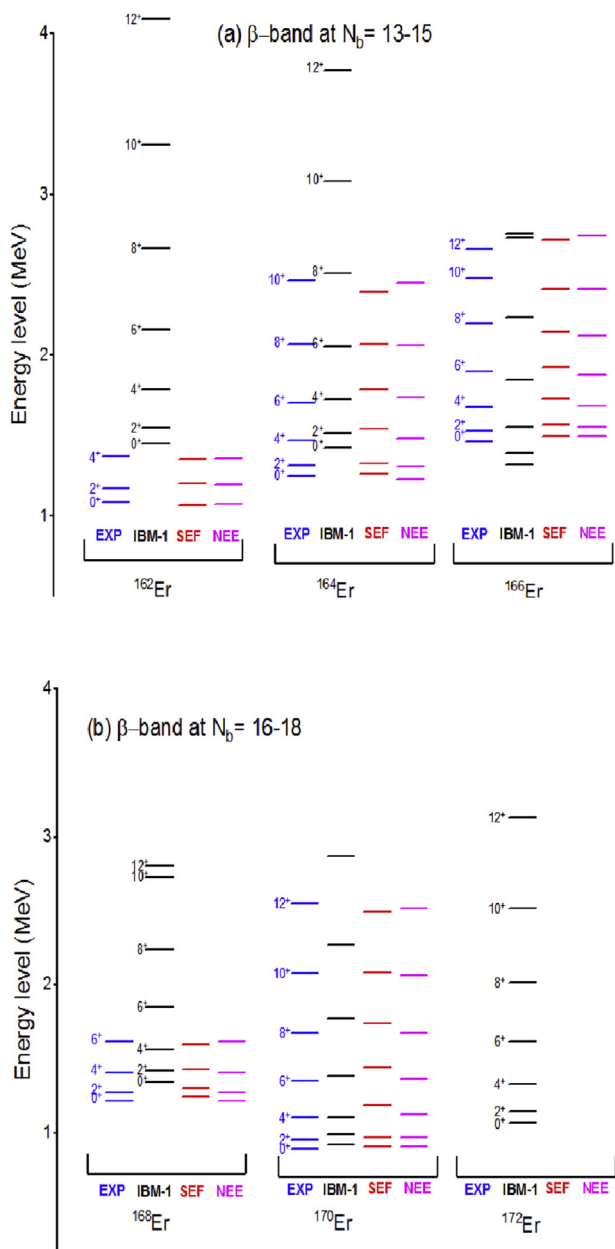


Fig. 2. The calculated and experimental values [27–32] of β -bands for the $^{162-172}\text{Er}$ isotopes.

obvious. Our results indicate that the ground state energy level calculations using the NEE formula are better represented compared to the IBM-1 and SEF calculations. The IBM-1 calculations for the ^{162}Er isotope revealed a maximum RMS error. The highest RMSD in SEF calculations is about 0.083 for the same isotopes. However, NEE calculations have the smallest RMS errors. In contrast to IBM-1 and SEF estimates, these errors decrease as excited energy levels increase.

The RMS errors in the IBM-1 model for both ^{162}Er and ^{164}Er isotopes are high compared to SEF and

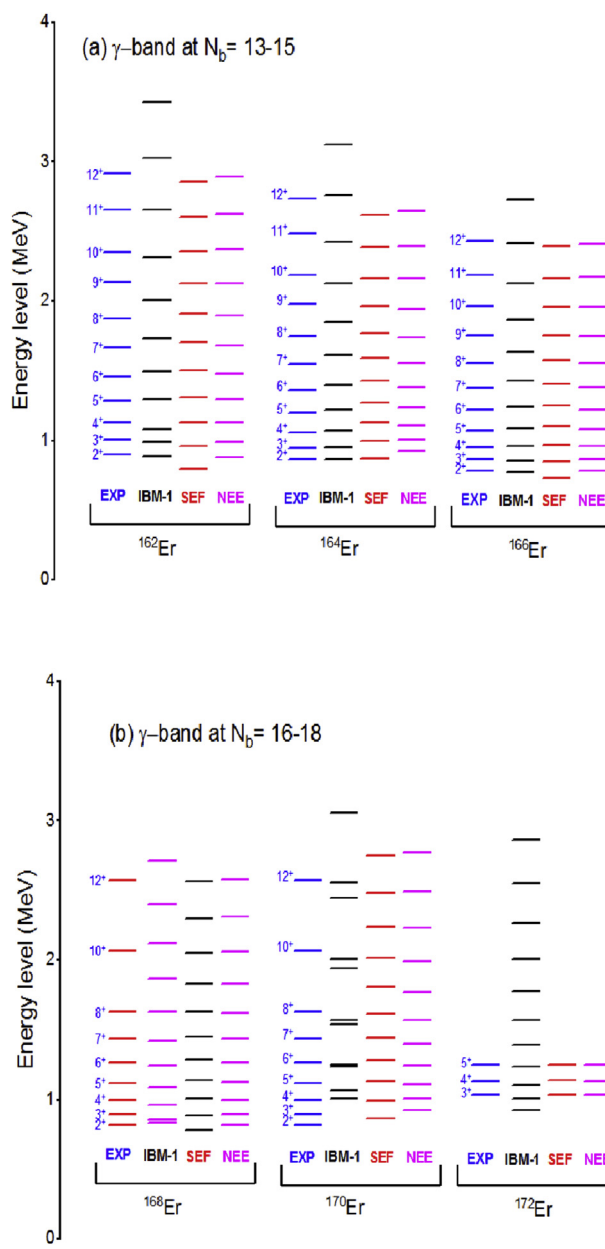


Fig. 3. The calculated and experimental values [27–32] of γ -bands for the $^{162-172}\text{Er}$ isotopes.

NEE calculations for the β -bands, as illustrated in Figure 2. The spacing between states in the ^{166}Er isotope is comparable for SEF and NEE calculations, but is greater for IBM-1 calculations. In the case of the ^{168}Er , the NEE calculation agrees perfectly with the experimental results compared to SEF and IBM-1 estimations. The RMS error in NEE calculations for the ^{170}Er isotope is also lower than SEF and IBM-1. Finally, the SEF and NEE for the ^{172}Er could not be calculated due to a lack of experimental data.

The SEF and NEE formulas are used to calculate the energy states of the γ -band for each isotope, as

shown in Figure 3. The largest RMS error for the ^{162}Er isotope in IBM-1 calculations is about 0.2242. Nonetheless, the SEF and NEE calculations are in line with the literature, particularly the NEE calculations. The RMS error for the ^{164}Er isotope is nearly identical to those for the SEF and NEE calculations and is less than the ratio in the IBM-1 calculation when compared to the experimental values. In contrast to the IBM-1 calculation, the NEE and SEF calculations for the ^{166}Er were almost identical to the experimental values. The NEE calculations are very close to experimental values compared to the IBM-1 and SEF calculations for the ^{168}Er isotope. We also observe that the spacing rates in the IBM-1 computation are significantly larger than in the SEF calculations but lower than in the NEE calculations for both the ^{170}Er and ^{172}Er isotopes.

4. Conclusions

This work used IBM-1, SEF, and NEE to find the energy levels of the ground state, β , and γ bands for $^{162-172}\text{Er}$ isotopes. The results of the GSB, β , and γ -bands revealed that IBM-1, SEF, NEE, and the available experimental data are all in agreement with some variations. Moreover, the NEE results for GSB, β , and γ -bands are more consistent with experimental results than the IBM-1 and SEF estimations. Furthermore, the $^{162-172}\text{Er}$ nuclei have a SU(3) transition limit.

Conflict of interest

No conflict of interest.

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